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An Experimental Performance Analysis for the Rate of Convergence of 5-Point Star on General Domains

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AN EXPERIMENTAL PERFORMANCE ANALYSIS FOR THE RATE OF CONVERGENCE OF 5-POINT STAR ON GENERAL DOMAINS

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ABSTRACT

This paper presents a systematic experimental performance analysis for the rate of convergence of *5-point star* discretization of elliptic partial differential equations on general domains. Twenty domains are selected for the experiment from the population of PDE's on nonrectangular domains in [7], including one rectangle for comparison. The experiment is made by using the Performance Evaluation System (PES) of ELLPACK [1], [6]. The result shows that the convergence of the ELLPACK module 5-POINT STAR behaves as $O(h^2)$, as predicted theoretically, on all 19 of the nonrectangular domains. This set includes a large variety of nonrectangular domains (only two have reentrant corners). We conclude that, with very high probability, this 5-POINT STAR module has $O(h^2)$ convergence on general domains.

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CONTENTS

1. INTRODUCTION

The *5-point star* is a classical, well investigated second order method for solving elliptic PDE's (see [2], [3], [4], [5] and references listed there). ELLPACK implements it in two versions, corresponding to rectangular and general domains, with the same name 5-POINT STAR. When the domain is rectangular the latter reduces to the former. It is known [2], for the Dirichlet problem for Poisson's equation, that the order of convergence of the method is $O(h^2)$ as long as the solution is smooth enough even though at points near the boundary the finite difference operator approximates the Laplace operator only to $O(h)$, which results in fact in the contribution to the truncation error of $O(h^3)$. However, we have not seen any systematic performance analysis for this method yet. This paper makes an extensive experimental study of the convergence rate of this *5-point star* method for general domains and presents data to confirm the $O(h^2)$ convergence performance of our ELLPACK module 5-POINT STAR.

2. THE NONRECTANGULAR PDE POPULATION

Ribbens and Rice [7] present twenty five domains, each with two parameters, for the nonrectangular PDE population and incorporate them into ELLPACK. We use 20 of them along with the Poisson equation:

$$U_{xx} + U_{yy} = f(x,y) \quad in \ \Omega \tag{2.1}$$

with Dirichlet boundary condition:

$$U = g(x,y) \qquad on \ \partial\Omega \ ; \tag{2.2}$$

The right hand sides f and g in (2.1) and (2.2) are defined such that the true solution is a smooth function as in Problem 4 of the PDE population in Appendix A of [8]:

$$U(x,y) = 3e^{x+y}x(1-x)y(1-y) \tag{2.3}$$

See Appendix A in the report by Mu and Rice [6] for sample files defining the PDE problems.

We use the classical central finite difference algorithm as described in [8], which is the discretization module 5-POINT STAR in the ELLPACK system.

3. THE EXPERIMENT

3.a The PDE Problems Solved

We use the Performance Evaluation System (PES) of ELLPACK to make the experimental performance analysis for the rate of convergence of *5-point star* on general domains. Twenty problems with general domains from [7] are selected for use. One is actually defined on a rectangle and used for comparison. The true solutions are all the same and very smooth (see (2.3)). Most domains have no re-entrant corners and some of them are rather complicated. They are of reasonable generality and represent a large variety of nonrectangular domains. For their exact definitions readers are referred to [7], their shapes are seen in Appendix A.2 along with the performance plots.

The problems chosen are solved by the double precision version ELLPACK [8] on a VAX 8600. The discretization, indexing and solution modules are 5-POINT STAR, AS IS and BAND GE, respectively.

Twelve levels of uniform grids of size 7×7, 9×9, 11×11, 13×13, 15×15, 17×17, 19×19, 21×21, 23×23, 25×25, 27×27 and 29×29 are first used on which all 20 problems are solved. Among the 240 ELLPACK jobs thus created, there are 4 failed runs for Problem 500-27 with grid 29×29 and Problem 500-71 with grids 9×9, 23×23 and 27×27. The reason is fatal error in the ellpack module DOMAIN PROCESSOR. This usually occurs when the grid is too coarse or there is an unfortunate location of one of the grid lines relative to the domain boundary.

For 9 of the problems where the results were not yet conclusive we generated from 2 to 26 more grids until the rate of convergence was "apparent" with reasonable accuracy. A total of 70 additional ELLPACK runs were made, the finest grids used are 111×111.

3.b Analysis of the Rates of Convergence

The performance analysis tool CONV2 is used to analyze the rate of convergence. It estimates the order of convergence by

$$conv2mx_n = \frac{\log\dfrac{rerr_1}{rerr_n}}{\log\dfrac{h_1}{h_n}} \tag{3.1}$$

where h is defined as

$$h = \frac{1}{ngridx-1}, \tag{3.2}$$

and *ngridx* is the number of x-grid points. Similarly *conv2mm* and *conv2l2* are defined with *rerr* substituted by *rerrmax* and *errl2*, respectively. Above, *rerr*, *rerrmax* stand for the relative errors of the current approximation in maximum norms on the current and 20

$\times$ 20 grid levels, respectively, while *errl2* is the approximation error in the discrete L_2 norm on the current grid as defined in [8]. All of these estimates are defined as zero on the coarsest grid. See [6] for the discussion of properties of CONV2.

For our 20 nonrectangular test problems, the output from CONV2 is given in Appendix A.1. In addition, we include in Appendix A.2 the plots of slopes of log(*err-nodes*) versus log(*nx*) where *err-nodes* and *nx* denote *rerrmax* and *ngridx-1*, respectively. The shapes of the domains are also given in Appendix A.2. The data suggest the following

1. The estimates for the convergence rate generated by *conv2mm* in CONV2 are obviously unreasonably larger than the theoretical value 2 in most of cases. This is due to the error in *rerrmax* caused by the interpolation from the coarsest grid to the 20×20 grid.
2. The estimated convergence rate of 5-POINT STAR measured by *conv2mx* and *conv2l2* in CONV2 has the following distribution. About 72% lie in the interval [1.5, 2.5], 21.4% in (2.5, +∞) and 6.6% in (-∞, 1.5).
3. This tool is useful to quantify the convergence rates observed, but too simple to capture the full complexity of the behavior of the error as the grid is refined. We prefer instead to rely on a visual analysis of plots of *err-nodes* versus *ngridx-1* on a *log-log* scale. Convergence of exactly order 2 results in a straight line with negative slope of -2. Almost exact $O(h^2)$ convergence is seen on some cases, but most error plots show some "random" variation due to unpredictable interaction between the domain shape and the grid location. We have collected enough data on each problem so that the theoretically expected $O(h^2)$ convergence can be observed from the plots.

4. CONCLUSIONS

From the above observation we conclude that

1. The program CONV2 is reliable for estimating convergence rates, except for *conv2mm*. when the initial grid is not fine enough.
2. The convergence of the 5-POINT STAR module behaves as $O(h^2)$ for general domains as predicted theoretically.

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APPENDIX A. The Experimental Data

1. Errors and convergence rates measured

500 1 b4p2unix/1/14/46/ (a=1,b=.8,c=.6)

nx	ny	h	nunk	rerr	rerrmax	errl2	conv2mx	conv2mm	conv2l2	solmax
7	7	1.3e-01	25	2.0e-03	6.6e-03	5.2e-04	0.0e+00	0.0e+00	0.0e+00	5.7e-01
9	9	1.0e-01	49	1.1e-03	3.2e-03	3.1e-04	1.9e+00	2.6e+00	1.8e+00	5.7e-01
11	11	8.0e-02	81	7.3e-04	1.7e-03	2.0e-04	1.9e+00	2.7e+00	1.9e+00	5.7e-01
13	13	6.7e-02	121	5.1e-04	1.1e-03	1.4e-04	1.9e+00	2.6e+00	1.9e+00	5.7e-01
15	15	5.7e-02	169	3.7e-04	6.4e-04	1.1e-04	1.9e+00	2.8e+00	1.9e+00	5.7e-01
17	17	5.0e-02	225	2.9e-04	4.9e-04	8.2e-05	2.0e+00	2.7e+00	1.9e+00	5.7e-01
19	19	4.4e-02	289	2.3e-04	3.7e-04	6.5e-05	2.0e+00	2.6e+00	1.9e+00	5.7e-01
21	21	4.0e-02	361	1.8e-04	2.5e-04	5.3e-05	2.0e+00	2.7e+00	1.9e+00	5.7e-01
23	23	3.6e-02	441	1.5e-04	2.1e-04	4.4e-05	2.0e+00	2.6e+00	1.9e+00	5.7e-01
25	25	3.3e-02	529	1.3e-04	2.0e-04	3.7e-05	2.0e+00	2.5e+00	1.9e+00	5.7e-01
27	27	3.1e-02	625	1.1e-04	1.6e-04	3.2e-05	2.0e+00	2.5e+00	1.9e+00	5.7e-01
29	29	2.9e-02	729	9.4e-05	1.1e-04	2.8e-05	2.0e+00	2.6e+00	1.9e+00	5.7e-01

500 4 b4p2unix/1/14/46/ (a=2,b=1.,c=1.)

nx	ny	h	nunk	rerr	rerrmax	errl2	conv2mx	conv2mm	conv2l2	solmax
7	7	1.7e-01	10	4.8e-03	2.8e-02	1.2e-03	0.0e+00	0.0e+00	0.0e+00	5.6e-01
9	9	1.3e-01	21	2.8e-03	1.6e-02	6.9e-04	1.9e+00	2.0e+00	1.8e+00	5.7e-01
11	11	1.0e-01	36	1.8e-03	9.8e-03	4.6e-04	2.0e+00	2.0e+00	1.8e+00	5.7e-01
13	13	8.3e-02	55	1.3e-03	6.0e-03	3.2e-04	1.9e+00	2.2e+00	1.9e+00	5.7e-01
15	15	7.1e-02	78	9.1e-04	4.4e-03	2.4e-04	2.0e+00	2.2e+00	1.9e+00	5.7e-01
17	17	6.3e-02	105	7.0e-04	2.7e-03	1.9e-04	2.0e+00	2.4e+00	1.9e+00	5.7e-01
19	19	5.6e-02	136	5.5e-04	1.2e-03	1.5e-04	2.0e+00	2.9e+00	1.9e+00	5.7e-01
21	21	5.0e-02	171	4.5e-04	9.3e-04	1.2e-04	2.0e+00	2.8e+00	1.9e+00	5.7e-01
23	23	4.5e-02	210	3.7e-04	1.2e-03	1.0e-04	2.0e+00	2.4e+00	1.9e+00	5.7e-01
25	25	4.2e-02	253	3.1e-04	9.6e-04	8.5e-05	2.0e+00	2.4e+00	1.9e+00	5.7e-01
27	27	3.8e-02	300	2.7e-04	7.9e-04	7.3e-05	2.0e+00	2.4e+00	1.9e+00	5.7e-01
29	29	3.6e-02	351	2.3e-04	6.9e-04	6.3e-05	2.0e+00	2.4e+00	1.9e+00	5.7e-01

500 5 b4p2unix/1/14/46/ (a=3,b=2.,c=1.)

nx	ny	h	nunk	rerr	rerrmax	errl2	conv2mx	conv2mm	conv2l2	solmax
7	7	6.7e-01	25	3.3e-02	1.5e+00	4.2e-02	0.0e+00	0.0e+00	0.0e+00	2.8e+00
9	9	5.0e-01	45	1.6e-02	4.5e-01	2.5e-02	2.6e+00	4.2e+00	1.8e+00	4.5e+00
11	11	4.0e-01	69	8.6e-03	1.4e-01	2.1e-02	2.6e+00	4.7e+00	1.4e+00	7.5e+00
13	13	3.3e-01	109	4.6e-03	6.0e-02	1.3e-02	2.9e+00	4.6e+00	1.7e+00	8.0e+00
15	15	2.9e-01	145	3.2e-03	4.4e-02	9.7e-03	2.8e+00	4.2e+00	1.7e+00	8.1e+00
17	17	2.5e-01	193	2.6e-03	1.3e-02	7.8e-03	2.6e+00	4.9e+00	1.7e+00	8.0e+00
19	19	2.2e-01	249	1.9e-03	5.4e-03	6.0e-03	2.6e+00	5.1e+00	1.8e+00	1.0e+01
21	21	2.0e-01	305	1.5e-03	2.4e-03	5.2e-03	2.6e+00	5.4e+00	1.7e+00	9.8e+00
23	23	1.8e-01	373	1.1e-03	5.1e-03	4.0e-03	2.6e+00	4.4e+00	1.8e+00	9.5e+00
25	25	1.7e-01	437	1.1e-03	5.0e-03	3.6e-03	2.5e+00	4.1e+00	1.8e+00	9.1e+00
27	27	1.5e-01	517	7.8e-04	2.2e-03	3.1e-03	2.6e+00	4.4e+00	1.8e+00	1.1e+01
29	29	1.4e-01	609	6.8e-04	1.9e-03	2.6e-03	2.5e+00	4.3e+00	1.8e+00	1.0e+01

500 15 b4p2unix/1/14/46/ (a=7,b=.4,c=.2)

nx	ny	h	nunk	rerr	rerrmax	errl2	conv2mx	conv2mm	conv2l2	solmax

7	7	1.4e-01	19	4.6e-04	4.3e-02	1.1e-04	0.0e+00	0.0e+00	0.0e+00	4.1e-01
9	9	1.0e-01	40	3.3e-04	2.6e-02	7.2e-05	1.2e+00	1.8e+00	1.5e+00	4.3e-01
11	11	8.3e-02	65	2.8e-04	7.8e-03	4.8e-05	9.9e-01	3.4e+00	1.6e+00	4.4e-01
13	13	6.9e-02	98	2.3e-04	2.5e-03	3.5e-05	9.9e-01	4.1e+00	1.7e+00	4.5e-01
15	15	6.0e-02	135	1.9e-04	1.1e-03	2.5e-05	1.1e+00	4.3e+00	1.7e+00	4.5e-01
17	17	5.2e-02	182	1.5e-04	5.2e-04	1.9e-05	1.1e+00	4.5e+00	1.8e+00	4.5e-01
19	19	4.6e-02	231	1.2e-04	2.8e-04	1.5e-05	1.2e+00	4.6e+00	1.8e+00	4.6e-01
21	21	4.2e-02	292	9.7e-05	1.9e-04	1.2e-05	1.3e+00	4.5e+00	1.9e+00	4.6e-01
23	23	3.8e-02	353	7.8e-05	1.4e-04	9.4e-06	1.4e+00	4.4e+00	1.9e+00	4.6e-01
25	25	3.5e-02	426	6.3e-05	1.0e-04	7.7e-06	1.4e+00	4.4e+00	1.9e+00	4.6e-01
27	27	3.2e-02	501	5.1e-05	6.8e-05	6.4e-06	1.5e+00	4.4e+00	1.9e+00	4.6e-01
29	29	3.0e-02	588	4.1e-05	6.0e-05	5.4e-06	1.6e+00	4.3e+00	2.0e+00	4.6e-01
45	45	1.9e-02	1492	1.1e-05	2.3e-05	2.2e-06	1.9e+00	3.8e+00	2.0e+00	4.7e-01
55	55	1.5e-02	2092	7.3e-06	1.4e-05	1.5e-06	1.9e+00	3.7e+00	1.9e+00	4.6e-01
65	65	1.3e-02	2987	4.7e-06	6.9e-06	1.0e-06	1.9e+00	3.7e+00	2.0e+00	4.7e-01

500 17 b4p2unix/1/14/46/ (a=8,b=1.,c=1.)

nx	ny	h	nunk	rerr	rerrmax	errl2	conv2mx	conv2mm	conv2l2	solmax
7	7	3.5e-01	25	5.6e-03	5.1e-02	3.3e-01	0.0e+00	0.0e+00	0.0e+00	1.2e+02
9	9	2.6e-01	45	7.7e-03	2.7e-02	2.0e-01	-1.1e+00	2.2e+00	1.7e+00	8.8e+01
11	11	2.1e-01	69	3.3e-03	2.8e-02	1.2e-01	1.0e+00	1.2e+00	2.0e+00	7.9e+01
13	13	1.7e-01	101	1.8e-03	4.1e-02	9.9e-02	1.6e+00	3.1e-01	1.8e+00	1.2e+02
15	15	1.5e-01	145	1.5e-03	4.8e-03	6.8e-02	1.5e+00	2.8e+00	1.9e+00	1.0e+02
17	17	1.3e-01	185	1.3e-03	3.2e-03	5.4e-02	1.5e+00	2.8e+00	1.9e+00	9.3e+01
19	19	1.2e-01	241	8.6e-04	2.0e-03	4.5e-02	1.7e+00	2.9e+00	1.8e+00	1.2e+02
21	21	1.0e-01	293	7.0e-04	1.2e-03	3.4e-02	1.7e+00	3.1e+00	1.9e+00	1.1e+02
23	23	9.4e-02	349	7.2e-04	2.4e-03	3.1e-02	1.6e+00	2.4e+00	1.8e+00	1.0e+02
25	25	8.7e-02	421	4.9e-04	1.6e-03	2.6e-02	1.8e+00	2.5e+00	1.8e+00	1.2e+02
27	27	8.0e-02	489	4.5e-04	1.4e-03	2.2e-02	1.7e+00	2.4e+00	1.8e+00	1.1e+02
29	29	7.4e-02	577	3.7e-04	3.3e-03	2.1e-02	1.8e+00	1.8e+00	1.8e+00	1.3e+02

500 21 b4p2unix/1/14/46/ (a=10,b=1.,c=0.)

nx	ny	h	nunk	rerr	rerrmax	errl2	conv2mx	conv2mm	conv2l2	solmax
7	7	6.9e-01	15	2.2e-02	8.9e-01	1.3e+00	0.0e+00	0.0e+00	0.0e+00	1.2e+02
9	9	5.2e-01	28	9.0e-03	3.8e-01	7.2e-01	3.2e+00	3.0e+00	2.0e+00	1.6e+02
11	11	4.2e-01	44	5.6e-03	2.2e-01	4.8e-01	2.7e+00	2.7e+00	1.9e+00	1.9e+02
13	13	3.5e-01	65	3.4e-03	1.3e-01	3.4e-01	2.7e+00	2.8e+00	1.9e+00	2.1e+02
15	15	3.0e-01	90	2.5e-03	6.4e-02	2.6e-01	2.6e+00	3.1e+00	1.9e+00	2.2e+02
17	17	2.6e-01	117	1.8e-03	2.5e-02	2.0e-01	2.5e+00	3.6e+00	1.9e+00	2.4e+02
19	19	2.3e-01	149	1.5e-03	4.7e-03	1.7e-01	2.5e+00	4.8e+00	1.8e+00	2.5e+02
21	21	2.1e-01	184	1.2e-03	5.6e-03	1.4e-01	2.4e+00	4.2e+00	1.8e+00	2.6e+02
23	23	1.9e-01	224	9.8e-04	5.4e-03	1.2e-01	2.4e+00	3.9e+00	1.8e+00	2.6e+02
25	25	1.7e-01	268	8.4e-04	6.6e-03	1.0e-01	2.4e+00	3.5e+00	1.8e+00	2.7e+02
27	27	1.6e-01	314	7.1e-04	6.7e-03	9.1e-02	2.3e+00	3.3e+00	1.8e+00	2.7e+02
29	29	1.5e-01	365	6.2e-04	6.1e-03	8.1e-02	2.3e+00	3.2e+00	1.8e+00	2.8e+02

1

500 22 b4p2unix/1/14/46/ (a=10,b=2.,c=0.)

nx	ny	h	nunk	rerr	rerrmax	errl2	conv2mx	conv2mm	conv2l2	solmax
7	7	6.9e-01	14	1.1e-02	9.2e-01	7.1e-01	0.0e+00	0.0e+00	0.0e+00	1.2e+02
9	9	5.2e-01	26	6.3e-03	3.8e-01	5.1e-01	2.0e+00	3.0e+00	1.1e+00	1.6e+02
11	11	4.2e-01	41	3.6e-03	2.2e-01	3.4e-01	2.2e+00	2.8e+00	1.5e+00	1.9e+02
13	13	3.5e-01	59	2.4e-03	1.3e-01	2.5e-01	2.2e+00	2.9e+00	1.5e+00	2.1e+02
15	15	3.0e-01	80	1.8e-03	6.4e-02	2.0e-01	2.1e+00	3.1e+00	1.5e+00	2.2e+02
17	17	2.6e-01	106	1.4e-03	2.5e-02	1.6e-01	2.1e+00	3.7e+00	1.5e+00	2.4e+02
19	19	2.3e-01	134	1.1e-03	6.4e-03	1.3e-01	2.1e+00	4.5e+00	1.6e+00	2.5e+02
21	21	2.1e-01	165	8.9e-04	8.4e-03	1.1e-01	2.1e+00	3.9e+00	1.6e+00	2.6e+02
23	23	1.9e-01	201	7.7e-04	6.7e-03	9.3e-02	2.1e+00	3.8e+00	1.6e+00	2.6e+02
25	25	1.7e-01	239	6.5e-04	6.6e-03	8.0e-02	2.1e+00	3.6e+00	1.6e+00	2.7e+02

27	27	1.6e-01	281	5.4e-04	6.7e-03	6.8e-02	2.1e+00	3.4e+00	1.6e+00	2.7e+02
29	29	1.5e-01	324	4.7e-04	6.1e-03	6.0e-02	2.1e+00	3.3e+00	1.6e+00	2.8e+02

500 25 b4p2unix/1/14/46/ (a=11,b=1.,c=0.)

nx	ny	h	nunk	rerr	rerrmax	errl2	conv2mx	conv2mm	conv212	solmax
7	7	7.8e-01	22	4.2e-02	3.7e-01	1.5e+04	0.0e+00	0.0e+00	0.0e+00	1.1e+06
9	9	5.8e-01	42	1.8e-02	2.1e-01	7.7e+03	2.9e+00	1.9e+00	2.4e+00	9.2e+05
11	11	4.7e-01	68	8.3e-03	3.7e-02	5.6e+03	3.2e+00	4.5e+00	2.0e+00	1.6e+06
13	13	3.9e-01	100	7.0e-03	2.7e-02	3.8e+03	2.6e+00	3.8e+00	2.0e+00	1.3e+06
15	15	3.3e-01	137	4.8e-03	3.1e-02	3.2e+03	2.6e+00	3.0e+00	1.9e+00	1.7e+06
17	17	2.9e-01	181	3.9e-03	5.4e-03	2.3e+03	2.4e+00	4.3e+00	1.9e+00	1.5e+06
19	19	2.6e-01	232	2.9e-03	7.7e-03	2.1e+03	2.4e+00	3.5e+00	1.8e+00	1.8e+06
21	21	2.3e-01	288	2.5e-03	2.3e-02	1.6e+03	2.3e+00	2.3e+00	1.9e+00	1.6e+06
23	23	2.1e-01	351	2.1e-03	3.9e-03	1.5e+03	2.3e+00	3.5e+00	1.8e+00	1.8e+06
25	25	1.9e-01	418	1.8e-03	1.0e-02	1.2e+03	2.3e+00	2.6e+00	1.9e+00	1.7e+06
27	27	1.8e-01	494	1.5e-03	6.3e-03	1.1e+03	2.3e+00	2.8e+00	1.8e+00	1.8e+06
29	29	1.7e-01	572	1.3e-03	8.6e-03	9.1e+02	2.2e+00	2.4e+00	1.8e+00	1.7e+06

500 27 b4p2unix/1/14/46/ (a=11,b=.5,c=0.)

nx	ny	h	nunk	rerr	rerrmax	errl2	conv2mx	conv2mm	conv212	solmax
7	7	6.2e-01	22	2.0e-02	4.6e-01	2.2e+01	0.0e+00	0.0e+00	0.0e+00	2.7e+03
9	9	4.6e-01	37	1.0e-02	2.2e-01	1.1e+01	2.3e+00	2.6e+00	2.6e+00	4.1e+03
11	11	3.7e-01	62	4.8e-03	1.6e-01	7.4e+00	2.8e+00	2.1e+00	2.1e+00	4.2e+03
13	13	3.1e-01	91	2.8e-03	4.8e-02	5.8e+00	2.8e+00	3.3e+00	1.9e+00	5.1e+03
15	15	2.6e-01	123	2.0e-03	5.7e-02	3.8e+00	2.7e+00	2.5e+00	2.1e+00	5.1e+03
17	17	2.3e-01	163	1.3e-03	2.8e-02	3.1e+00	2.8e+00	2.9e+00	2.0e+00	5.6e+03
19	19	2.1e-01	207	1.2e-03	1.9e-02	2.7e+00	2.6e+00	2.9e+00	1.9e+00	5.5e+03
21	21	1.9e-01	255	9.3e-04	1.8e-02	2.2e+00	2.6e+00	2.7e+00	1.9e+00	5.9e+03
23	23	1.7e-01	314	7.4e-04	8.1e-03	1.7e+00	2.5e+00	3.1e+00	2.0e+00	5.9e+03
25	25	1.6e-01	371	7.2e-04	8.0e-03	1.6e+00	2.4e+00	2.9e+00	1.9e+00	5.8e+03
27	27	1.4e-01	434	5.7e-04	6.9e-03	1.4e+00	2.4e+00	2.9e+00	1.9e+00	6.1e+03
45	45	8.4e-02	1263	2.1e-04	9.9e-04	5.2e-01	2.3e+00	3.1e+00	1.9e+00	6.5e+03
55	55	6.9e-02	1859	1.4e-04	3.4e-04	3.5e-01	2.3e+00	3.3e+00	1.9e+00	6.6e+03
65	65	5.8e-02	2633	9.8e-05	5.1e-04	2.5e-01	2.2e+00	2.9e+00	1.9e+00	6.7e+03
75	75	5.0e-02	3530	7.4e-05	9.4e-05	1.9e-01	2.2e+00	3.4e+00	1.9e+00	6.7e+03

500 28 b4p2unix/1/14/46/ (a=11,b=1.5,c=0.)

nx	ny	h	nunk	rerr	rerrmax	errl2	conv2mx	conv2mm	conv212	solmax
7	7	1.5e+00	22	9.5e-02	1.2e+00	2.9e+07	0.0e+00	0.0e+00	0.0e+00	8.2e+08
9	9	1.2e+00	39	3.7e-02	8.2e-01	1.7e+07	3.3e+00	1.4e+00	1.9e+00	9.8e+08
11	11	9.3e-01	62	3.6e-02	5.4e-01	1.5e+07	1.9e+00	1.6e+00	1.3e+00	1.1e+09
13	13	7.7e-01	91	1.5e-02	1.5e-01	1.4e+07	2.7e+00	3.0e+00	1.1e+00	2.5e+09
15	15	6.6e-01	125	1.2e-02	1.0e-01	1.0e+07	2.4e+00	2.9e+00	1.2e+00	2.3e+09
17	17	5.8e-01	163	9.5e-03	4.3e-02	7.7e+06	2.3e+00	3.4e+00	1.3e+00	2.1e+09
19	19	5.1e-01	209	7.5e-03	5.2e-02	6.0e+06	2.3e+00	2.9e+00	1.4e+00	2.0e+09
21	21	4.6e-01	257	6.6e-03	4.9e-02	5.0e+06	2.2e+00	2.7e+00	1.5e+00	2.0e+09
23	23	4.2e-01	314	6.5e-03	5.4e-02	4.5e+06	2.1e+00	2.4e+00	1.4e+00	1.9e+09
25	25	3.9e-01	374	4.5e-03	2.5e-02	4.0e+06	2.2e+00	2.8e+00	1.4e+00	2.5e+09
27	27	3.6e-01	439	3.9e-03	8.9e-03	3.4e+06	2.2e+00	3.3e+00	1.5e+00	2.4e+09
29	29	3.3e-01	510	3.5e-03	2.2e-02	2.9e+06	2.1e+00	2.6e+00	1.5e+00	2.3e+09

500 37 b4p2unix/1/14/46/ (a=15,b=.5,c=1.5)

nx	ny	h	nunk	rerr	rerrmax	errl2	conv2mx	conv2mm	conv212	solmax
7	7	2.6e-01	21	1.7e-02	4.5e-01	1.5e-02	0.0e+00	0.0e+00	0.0e+00	1.5e+00
9	9	1.9e-01	38	5.9e-03	1.4e-01	7.3e-03	3.7e+00	4.0e+00	2.4e+00	2.2e+00
11	11	1.6e-01	60	3.3e-03	2.9e-02	4.7e-03	3.2e+00	5.4e+00	2.2e+00	2.7e+00

13	13	1.3e-01	90	2.3e-03	1.4e-02	3.3e-03	2.9e+00	5.0e+00	2.2e+00	2.8e+00
15	15	1.1e-01	122	1.8e-03	3.7e-02	2.5e-03	2.6e+00	3.0e+00	2.1e+00	2.7e+00
17	17	9.7e-02	160	1.5e-03	5.7e-03	1.9e-03	2.5e+00	4.5e+00	2.1e+00	2.6e+00
19	19	8.7e-02	206	1.0e-03	4.7e-03	1.4e-03	2.6e+00	4.2e+00	2.1e+00	2.9e+00
21	21	7.8e-02	257	8.8e-04	2.1e-03	1.2e-03	2.5e+00	4.5e+00	2.1e+00	2.8e+00
23	23	7.1e-02	312	7.4e-04	7.3e-03	9.8e-04	2.4e+00	3.2e+00	2.1e+00	2.8e+00
25	25	6.5e-02	371	6.2e-04	3.7e-03	8.3e-04	2.4e+00	3.5e+00	2.1e+00	2.8e+00
27	27	6.0e-02	438	4.9e-04	1.6e-03	7.0e-04	2.4e+00	3.8e+00	2.1e+00	3.0e+00
29	29	5.6e-02	509	4.3e-04	2.2e-03	6.0e-04	2.4e+00	3.5e+00	2.1e+00	3.0e+00

500 43 b4p2unix/1/14/46/ (a=18,b=0.,c=0.)

nx	ny	h	nunk	rerr	rerrmax	errl2	conv2mx	conv2mm	conv212	solmax
7	7	8.7e-01	19	7.5e-02	2.5e+00	1.0e+00	0.0e+00	0.0e+00	0.0e+00	3.7e+01
9	9	6.5e-01	39	2.7e-02	5.7e-01	6.2e-01	3.5e+00	5.1e+00	1.8e+00	7.8e+01
11	11	5.2e-01	64	1.2e-02	1.4e-01	3.8e-01	3.5e+00	5.6e+00	2.0e+00	8.7e+01
13	13	4.3e-01	89	7.8e-03	1.8e-01	2.6e-01	3.3e+00	3.8e+00	2.0e+00	9.0e+01
15	15	3.7e-01	126	6.5e-03	1.6e-01	2.1e-01	2.9e+00	3.2e+00	1.9e+00	8.9e+01
17	17	3.2e-01	163	5.0e-03	2.9e-02	1.6e-01	2.7e+00	4.5e+00	1.9e+00	8.7e+01
19	19	2.9e-01	204	4.0e-03	3.3e-02	1.2e-01	2.7e+00	3.9e+00	2.0e+00	8.4e+01
21	21	2.6e-01	253	3.3e-03	6.7e-02	1.0e-01	2.6e+00	3.0e+00	1.9e+00	8.8e+01
23	23	2.4e-01	309	2.9e-03	6.8e-02	9.6e-02	2.5e+00	2.8e+00	1.8e+00	9.0e+01
25	25	2.2e-01	365	2.4e-03	2.8e-02	7.9e-02	2.5e+00	3.2e+00	1.8e+00	9.0e+01
27	27	2.0e-01	433	1.7e-03	1.3e-02	6.6e-02	2.6e+00	3.6e+00	1.9e+00	1.1e+02
29	29	1.9e-01	501	1.6e-03	2.5e-02	6.1e-02	2.5e+00	3.0e+00	1.8e+00	1.1e+02
45	45	1.2e-01	1240	7.0e-04	5.7e-03	2.5e-02	2.3e+00	3.1e+00	1.9e+00	1.0e+02
55	55	9.6e-02	1839	4.4e-04	5.3e-04	1.7e-02	2.3e+00	3.9e+00	1.9e+00	1.1e+02
65	65	8.1e-02	2598	3.2e-04	8.4e-04	1.2e-02	2.3e+00	3.4e+00	1.9e+00	1.1e+02
75	75	7.0e-02	3482	2.3e-04	1.7e-03	9.4e-03	2.3e+00	2.9e+00	1.9e+00	1.1e+02

1

500 53 b4p2unix/1/14/46/ (a=21,b=.2,c=.5)

nx	ny	h	nunk	rerr	rerrmax	errl2	conv2mx	conv2mm	conv212	solmax
7	7	3.5e-01	16	8.2e-03	4.2e-01	6.1e-03	0.0e+00	0.0e+00	0.0e+00	1.4e+00
9	9	2.6e-01	29	2.4e-03	1.2e-01	3.8e-03	4.3e+00	4.3e+00	1.7e+00	2.6e+00
11	11	2.1e-01	46	2.5e-03	8.8e-02	2.2e-03	2.3e+00	3.0e+00	2.0e+00	1.8e+00
13	13	1.7e-01	68	9.7e-04	5.4e-02	1.5e-03	3.1e+00	3.0e+00	2.0e+00	2.7e+00
15	15	1.5e-01	92	7.7e-04	2.6e-02	1.1e-03	2.8e+00	3.3e+00	2.0e+00	2.7e+00
17	17	1.3e-01	121	6.0e-04	2.4e-02	8.4e-04	2.7e+00	2.9e+00	2.0e+00	2.6e+00
19	19	1.2e-01	151	4.8e-04	1.2e-02	6.5e-04	2.6e+00	3.3e+00	2.0e+00	2.4e+00
21	21	1.0e-01	188	3.5e-04	1.4e-02	5.2e-04	2.6e+00	2.8e+00	2.0e+00	2.6e+00
23	23	9.5e-02	225	3.1e-04	9.4e-03	4.4e-04	2.5e+00	2.9e+00	2.0e+00	2.5e+00
25	25	8.7e-02	270	2.5e-04	4.6e-03	3.6e-04	2.5e+00	3.2e+00	2.0e+00	2.7e+00
27	27	8.0e-02	318	2.1e-04	8.1e-03	3.1e-04	2.5e+00	2.7e+00	2.0e+00	2.7e+00
29	29	7.4e-02	369	1.8e-04	5.5e-03	2.7e-04	2.5e+00	2.8e+00	2.0e+00	2.7e+00

500 55 b4p2unix/1/14/46/ (a=21,b=20.,c=.6)

nx	ny	h	nunk	rerr	rerrmax	errl2	conv2mx	conv2mm	conv212	solmax
7	7	3.6e-01	17	1.0e-02	3.8e-01	8.1e-03	0.0e+00	0.0e+00	0.0e+00	1.5e+00
9	9	2.7e-01	31	3.3e-03	2.1e-01	3.6e-03	3.9e+00	2.1e+00	2.8e+00	2.1e+00
11	11	2.2e-01	51	1.7e-03	7.6e-02	2.3e-03	3.4e+00	3.2e+00	2.5e+00	2.9e+00
13	13	1.8e-01	73	1.8e-03	5.3e-02	1.4e-03	2.5e+00	2.9e+00	2.5e+00	2.1e+00
15	15	1.6e-01	100	1.1e-03	4.9e-02	1.0e-03	2.7e+00	2.4e+00	2.5e+00	2.6e+00
17	17	1.4e-01	122	6.4e-04	2.1e-02	8.5e-04	2.8e+00	3.0e+00	2.3e+00	2.7e+00
19	19	1.2e-01	157	5.0e-04	1.0e-02	7.1e-04	2.7e+00	3.3e+00	2.2e+00	3.1e+00
21	21	1.1e-01	198	3.6e-04	1.2e-02	5.7e-04	2.8e+00	2.9e+00	2.2e+00	3.1e+00
23	23	9.9e-02	240	3.0e-04	9.0e-03	4.6e-04	2.7e+00	2.9e+00	2.2e+00	2.9e+00
25	25	9.0e-02	286	2.6e-04	5.3e-03	3.8e-04	2.6e+00	3.1e+00	2.2e+00	2.9e+00
27	27	8.3e-02	338	2.3e-04	4.9e-03	3.3e-04	2.6e+00	3.0e+00	2.2e+00	2.8e+00
29	29	7.8e-02	393	1.9e-04	6.2e-03	2.8e-04	2.6e+00	2.7e+00	2.2e+00	3.0e+00

500 57 b4p2unix/1/14/46/ (a=22,b=1.,c=.1)

nx	ny	h	nunk	rerr	rerrmax	errl2	conv2mx	conv2mm	conv2l2	solmax
7	7	4.5e-01	14	1.8e-02	2.5e+00	1.7e-01	0.0e+00	0.0e+00	0.0e+00	3.2e+01
9	9	3.4e-01	23	7.9e-03	2.1e-01	1.1e-01	2.9e+00	8.7e+00	1.7e+00	6.0e+01
11	11	2.7e-01	37	3.9e-03	1.5e-02	6.3e-02	3.0e+00	1.0e+01	2.0e+00	8.6e+01
13	13	2.2e-01	53	2.1e-03	1.3e-02	3.9e-02	3.1e+00	7.6e+00	2.2e+00	1.1e+02
15	15	1.9e-01	73	1.3e-03	1.1e-02	2.4e-02	3.1e+00	6.4e+00	2.3e+00	1.3e+02
17	17	1.7e-01	93	7.9e-04	3.5e-03	1.7e-02	3.2e+00	6.7e+00	2.4e+00	1.4e+02
19	19	1.5e-01	118	5.2e-04	9.9e-04	1.1e-02	3.2e+00	7.1e+00	2.5e+00	1.6e+02
20	20	1.4e-01	132	4.3e-04	4.3e-04	9.5e-03	3.3e+00	7.5e+00	2.5e+00	1.6e+02
21	21	1.4e-01	146	3.5e-04	6.9e-03	8.0e-03	3.3e+00	4.9e+00	2.6e+00	1.7e+02
23	23	1.2e-01	177	2.3e-04	5.4e-03	5.4e-03	3.4e+00	4.7e+00	2.7e+00	1.8e+02
25	25	1.1e-01	209	1.6e-04	2.5e-03	4.5e-03	3.4e+00	5.0e+00	2.6e+00	1.8e+02
27	27	1.0e-01	247	1.1e-04	2.4e-03	3.5e-03	3.5e+00	4.7e+00	2.7e+00	1.9e+02
29	29	9.7e-02	285	7.7e-05	2.1e-03	3.2e-03	3.5e+00	4.6e+00	2.6e+00	2.0e+02
30	30	9.3e-02	307	6.5e-05	9.6e-04	2.9e-03	3.6e+00	5.0e+00	2.6e+00	2.0e+02
31	31	9.0e-02	328	6.1e-05	7.6e-04	2.6e-03	3.5e+00	5.0e+00	2.6e+00	2.0e+02
33	33	8.4e-02	374	5.8e-05	5.3e-04	2.4e-03	3.4e+00	5.1e+00	2.6e+00	2.1e+02
35	35	8.0e-02	425	5.4e-05	3.5e-04	2.1e-03	3.4e+00	5.1e+00	2.6e+00	2.1e+02
37	37	7.5e-02	472	7.1e-05	2.1e-03	1.9e-03	3.1e+00	4.0e+00	2.5e+00	1.6e+02
39	39	7.1e-02	525	6.6e-05	6.4e-05	1.7e-03	3.0e+00	5.7e+00	2.5e+00	1.6e+02
40	40	6.9e-02	554	6.3e-05	4.8e-04	1.6e-03	3.0e+00	4.6e+00	2.5e+00	1.6e+02
41	41	6.8e-02	581	6.0e-05	9.0e-04	1.6e-03	3.0e+00	4.2e+00	2.5e+00	1.7e+02
43	43	6.4e-02	644	5.4e-05	1.4e-03	1.4e-03	3.0e+00	3.8e+00	2.5e+00	1.7e+02
45	45	6.2e-02	704	4.8e-05	1.6e-03	1.3e-03	3.0e+00	3.7e+00	2.5e+00	1.8e+02
49	49	5.6e-02	841	3.9e-05	2.5e-04	1.1e-03	3.0e+00	4.4e+00	2.5e+00	1.8e+02
53	53	5.2e-02	986	3.2e-05	1.3e-04	9.0e-04	2.9e+00	4.6e+00	2.5e+00	1.8e+02
57	57	4.8e-02	1143	2.5e-05	2.8e-05	7.7e-04	3.0e+00	5.1e+00	2.4e+00	1.9e+02
61	61	4.5e-02	1315	1.9e-05	8.0e-05	6.7e-04	3.0e+00	4.5e+00	2.4e+00	2.0e+02
65	65	4.2e-02	1493	1.4e-05	1.3e-04	5.9e-04	3.0e+00	4.2e+00	2.4e+00	2.0e+02
69	69	4.0e-02	1687	1.0e-05	6.8e-05	5.2e-04	3.1e+00	4.3e+00	2.4e+00	2.1e+02
73	73	3.8e-02	1890	8.7e-06	4.1e-05	4.7e-04	3.1e+00	4.4e+00	2.4e+00	2.1e+02
77	77	3.6e-02	2107	7.6e-06	7.1e-06	4.2e-04	3.1e+00	5.0e+00	2.4e+00	2.2e+02
81	81	3.4e-02	2335	7.0e-06	3.5e-05	3.8e-04	3.0e+00	4.3e+00	2.4e+00	2.2e+02
85	85	3.2e-02	2571	6.4e-06	6.6e-05	3.5e-04	3.0e+00	4.0e+00	2.4e+00	2.3e+02
93	93	2.9e-02	3086	5.9e-06	1.8e-05	2.9e-04	2.9e+00	4.3e+00	2.3e+00	2.3e+02
101	101	2.7e-02	3648	4.8e-06	2.4e-05	2.5e-04	2.9e+00	4.1e+00	2.3e+00	2.1e+02
105	105	2.6e-02	3947	4.3e-06	4.0e-05	2.3e-04	2.9e+00	3.9e+00	2.3e+00	2.1e+02
109	109	2.5e-02	4257	3.8e-06	1.3e-05	2.1e-04	2.9e+00	4.2e+00	2.3e+00	2.2e+02
111	111	2.5e-02	4413	3.6e-06	1.2e-05	2.1e-04	2.9e+00	4.2e+00	2.3e+00	2.2e+02

500 59 b4p2unix/1/14/46/ (a=22,b=.8,c=.8)

nx	ny	h	nunk	rerr	rerrmax	errl2	conv2mx	conv2mm	conv2l2	solmax
7	7	4.6e-01	3	3.2e-03	4.2e+00	2.6e-01	0.0e+00	0.0e+00	0.0e+00	8.7e+01
9	9	3.4e-01	14	1.8e-03	9.6e-01	1.9e-01	1.9e+00	5.1e+00	1.0e+00	2.3e+02
11	11	2.7e-01	23	2.0e-03	1.5e-01	2.1e-01	9.0e-01	6.5e+00	4.1e-01	3.8e+02
13	13	2.3e-01	35	1.5e-03	1.4e-01	1.8e-01	1.1e+00	4.9e+00	5.2e-01	5.4e+02
15	15	2.0e-01	47	1.0e-03	4.3e-02	1.4e-01	1.3e+00	5.4e+00	7.1e-01	6.8e+02
17	17	1.7e-01	63	7.2e-04	4.3e-02	1.0e-01	1.5e+00	4.7e+00	9.3e-01	8.0e+02
19	19	1.5e-01	78	5.1e-04	8.2e-03	7.7e-02	1.7e+00	5.7e+00	1.1e+00	9.1e+02
21	21	1.4e-01	96	3.7e-04	4.6e-03	5.9e-02	1.8e+00	5.6e+00	1.2e+00	1.0e+03
23	23	1.3e-01	113	2.6e-04	7.5e-03	4.5e-02	1.9e+00	4.9e+00	1.3e+00	1.1e+03
25	25	1.1e-01	137	1.9e-04	4.8e-03	3.5e-02	2.0e+00	4.9e+00	1.4e+00	1.2e+03
27	27	1.0e-01	163	1.3e-04	4.2e-03	2.9e-02	2.2e+00	4.7e+00	1.5e+00	1.3e+03
29	29	9.8e-02	186	9.0e-05	4.1e-03	2.4e-02	2.3e+00	4.5e+00	1.5e+00	1.3e+03
31	31	9.1e-02	216	5.7e-05	5.9e-03	2.0e-02	2.5e+00	4.1e+00	1.6e+00	1.4e+03
33	33	8.6e-02	245	5.4e-05	3.2e-03	1.7e-02	2.4e+00	4.3e+00	1.6e+00	1.4e+03
35	35	8.1e-02	272	5.1e-05	1.0e-03	1.5e-02	2.4e+00	4.8e+00	1.6e+00	1.5e+03
37	37	7.6e-02	305	5.5e-05	1.1e-03	1.4e-02	2.3e+00	4.6e+00	1.6e+00	1.3e+03

41	41	6.9e-02	382	5.6e-05	2.1e-03	1.1e-02	2.1e+00	4.0e+00	1.6e+00	1.2e+03
43	43	6.5e-02	417	5.1e-05	3.4e-03	1.0e-02	2.1e+00	3.7e+00	1.6e+00	1.2e+03
45	45	6.2e-02	458	4.6e-05	1.4e-03	9.5e-03	2.1e+00	4.0e+00	1.7e+00	1.3e+03
49	49	5.7e-02	550	3.6e-05	2.1e-04	7.7e-03	2.1e+00	4.8e+00	1.7e+00	1.3e+03
53	53	5.3e-02	645	3.0e-05	1.4e-04	6.4e-03	2.2e+00	4.8e+00	1.7e+00	1.4e+03
57	57	4.9e-02	743	2.5e-05	3.5e-05	5.6e-03	2.2e+00	5.2e+00	1.7e+00	1.5e+03
61	61	4.6e-02	859	2.1e-05	2.2e-04	4.8e-03	2.2e+00	4.3e+00	1.7e+00	1.5e+03
65	65	4.3e-02	980	1.7e-05	1.0e-04	4.1e-03	2.2e+00	4.5e+00	1.7e+00	1.6e+03
69	69	4.0e-02	1101	1.3e-05	3.7e-05	3.6e-03	2.3e+00	4.8e+00	1.8e+00	1.6e+03
73	73	3.8e-02	1233	9.8e-06	3.8e-05	3.2e-03	2.3e+00	4.7e+00	1.8e+00	1.6e+03
81	81	3.4e-02	1527	7.5e-06	7.3e-05	2.6e-03	2.3e+00	4.2e+00	1.8e+00	1.7e+03
85	85	3.3e-02	1677	7.0e-06	3.7e-05	2.3e-03	2.3e+00	4.4e+00	1.8e+00	1.7e+03
89	89	3.1e-02	1839	7.2e-06	2.0e-05	2.1e-03	2.3e+00	4.6e+00	1.8e+00	1.5e+03
93	93	3.0e-02	2018	6.3e-06	1.8e-05	2.0e-03	2.3e+00	4.5e+00	1.8e+00	1.6e+03
101	101	2.7e-02	2379	4.8e-06	2.8e-05	1.6e-03	2.3e+00	4.2e+00	1.8e+00	1.6e+03
109	109	2.5e-02	2774	3.7e-06	7.3e-06	1.4e-03	2.3e+00	4.6e+00	1.8e+00	1.7e+03

500 61 b4p2unix/1/14/46/ (a=23,b=0.,c=.5)

nx	ny	h	nunk	rerr	rerrmax	errl2	conv2mx	conv2mm	conv212	solmax
7	7	5.2e-01	16	1.6e-02	5.3e-01	1.5e+00	0.0e+00	0.0e+00	0.0e+00	2.3e+02
9	9	3.9e-01	30	9.5e-03	6.6e-01	7.5e-01	1.8e+00	-7.6e-01	2.4e+00	2.0e+02
11	11	3.1e-01	43	5.8e-03	7.2e-02	5.7e-01	1.9e+00	3.9e+00	1.9e+00	3.6e+02
13	13	2.6e-01	65	4.0e-03	5.8e-02	3.5e-01	2.0e+00	3.2e+00	2.1e+00	3.0e+02
15	15	2.2e-01	89	2.5e-03	2.0e-02	2.7e-01	2.2e+00	3.9e+00	2.0e+00	4.2e+02
17	17	1.9e-01	115	1.8e-03	2.7e-02	2.0e-01	2.2e+00	3.1e+00	2.1e+00	3.6e+02
19	19	1.7e-01	145	1.5e-03	1.8e-02	1.7e-01	2.1e+00	3.1e+00	2.0e+00	4.6e+02
21	21	1.6e-01	179	6.4e-04	5.6e-03	1.3e-01	2.7e+00	3.8e+00	2.0e+00	5.6e+02
23	23	1.4e-01	212	8.6e-04	1.0e-02	1.2e-01	2.2e+00	3.0e+00	2.0e+00	4.8e+02
25	25	1.3e-01	258	4.6e-04	7.9e-03	9.5e-02	2.5e+00	3.0e+00	2.0e+00	5.6e+02
27	27	1.2e-01	296	5.0e-04	4.3e-03	8.4e-02	2.3e+00	3.3e+00	2.0e+00	5.0e+02
29	29	1.1e-01	351	3.5e-04	4.5e-03	7.2e-02	2.5e+00	3.1e+00	2.0e+00	5.7e+02
45	45	7.1e-02	853	1.5e-04	7.2e-04	3.1e-02	2.3e+00	3.3e+00	1.9e+00	5.8e+02
55	55	5.8e-02	1287	1.1e-04	1.4e-04	2.1e-02	2.3e+00	3.8e+00	1.9e+00	5.3e+02
65	65	4.9e-02	1816	7.5e-05	1.9e-04	1.5e-02	2.3e+00	3.3e+00	1.9e+00	5.7e+02
75	75	4.2e-02	2420	5.4e-05	8.5e-05	1.2e-02	2.3e+00	3.5e+00	1.9e+00	6.0e+02

500 69 b4p2unix/1/14/46/ (a=25,b=.2,c=0.)

nx	ny	h	nunk	rerr	rerrmax	errl2	conv2mx	conv2mm	conv212	solmax
7	7	3.6e-01	20	1.1e-02	1.3e+00	5.6e-02	0.0e+00	0.0e+00	0.0e+00	1.1e+01
9	9	2.7e-01	37	1.9e-02	1.3e-01	7.9e-02	-1.8e+00	8.0e+00	-1.2e+00	1.9e+01
11	11	2.2e-01	58	6.0e-03	9.8e-01	2.2e-02	1.3e+00	5.7e-01	1.9e+00	1.1e+01
13	13	1.8e-01	82	6.0e-03	3.4e-02	2.5e-02	9.3e-01	5.3e+00	1.2e+00	2.4e+01
15	15	1.6e-01	111	2.8e-03	3.2e-02	1.1e-02	1.7e+00	4.4e+00	2.0e+00	1.8e+01
17	17	1.4e-01	147	2.6e-03	2.8e-02	1.1e-02	1.5e+00	3.9e+00	1.7e+00	2.7e+01
19	19	1.2e-01	185	1.5e-03	8.2e-03	6.0e-03	1.9e+00	4.6e+00	2.0e+00	2.2e+01
21	21	1.1e-01	230	1.4e-03	6.7e-03	5.8e-03	1.7e+00	4.4e+00	1.9e+00	2.9e+01
23	23	9.9e-02	273	8.6e-04	5.6e-03	4.2e-03	2.0e+00	4.2e+00	2.0e+00	2.5e+01
25	25	9.1e-02	326	8.7e-04	3.2e-03	3.8e-03	1.9e+00	4.3e+00	2.0e+00	3.1e+01
27	27	8.4e-02	382	5.8e-04	5.7e-03	3.0e-03	2.0e+00	3.7e+00	2.0e+00	2.7e+01
29	29	7.8e-02	446	5.7e-04	1.9e-03	2.7e-03	1.9e+00	4.2e+00	2.0e+00	3.2e+01
45	45	5.0e-02	1097	1.4e-04	5.1e-04	1.0e-03	2.2e+00	3.9e+00	2.0e+00	3.4e+01
55	55	4.1e-02	1646	8.5e-05	9.9e-05	7.0e-04	2.2e+00	4.3e+00	2.0e+00	3.3e+01
65	65	3.4e-02	2320	5.8e-05	9.7e-05	5.1e-04	2.2e+00	4.0e+00	2.0e+00	3.3e+01
75	75	3.0e-02	3099	4.0e-05	8.8e-05	3.8e-04	2.3e+00	3.8e+00	2.0e+00	3.4e+01

1

500 70 b4p2unix/1/14/46/ (a=25,b=.2,c=1.)

nx	ny	h	nunk	rerr	rerrmax	errl2	conv2mx	conv2mm	conv212	solmax
7	7	5.4e-01	19	6.5e-02	4.9e-01	3.3e-01	0.0e+00	0.0e+00	0.0e+00	1.7e+01

9	9	4.0e-01	34	2.6e-02	2.1e+00	6.3e-02	3.2e+00	-5.1e+00	5.7e+00	6.5e+00
11	11	3.2e-01	55	4.5e-02	6.3e-01	9.3e-02	7.2e-01	-4.9e-01	2.5e+00	1.1e+01
13	13	2.7e-01	78	9.1e-03	6.1e-02	5.1e-02	2.8e+00	3.0e+00	2.7e+00	3.0e+01
15	15	2.3e-01	105	3.9e-03	2.5e-02	2.2e-02	3.3e+00	3.5e+00	3.2e+00	2.0e+01
17	17	2.0e-01	136	8.8e-03	9.5e-02	2.6e-02	2.0e+00	1.7e+00	2.6e+00	2.1e+01
19	19	1.8e-01	173	4.4e-03	1.2e-02	1.7e-02	2.4e+00	3.4e+00	2.7e+00	2.5e+01
21	21	1.6e-01	215	2.5e-03	3.0e-02	1.1e-02	2.7e+00	2.3e+00	2.8e+00	2.0e+01
23	23	1.5e-01	258	3.4e-03	3.4e-02	1.1e-02	2.3e+00	2.0e+00	2.6e+00	2.6e+01
25	25	1.3e-01	308	1.9e-03	5.9e-03	8.5e-03	2.5e+00	3.2e+00	2.6e+00	3.0e+01
27	27	1.2e-01	361	1.2e-03	1.6e-02	6.5e-03	2.7e+00	2.3e+00	2.7e+00	2.5e+01
29	29	1.2e-01	418	1.7e-03	1.3e-02	6.2e-03	2.4e+00	2.3e+00	2.6e+00	3.0e+01
31	31	1.1e-01	480	1.0e-03	3.4e-03	5.2e-03	2.6e+00	3.1e+00	2.6e+00	3.3e+01
37	37	9.0e-02	689	7.0e-04	2.6e-03	3.6e-03	2.5e+00	2.9e+00	2.5e+00	3.0e+01
39	39	8.5e-02	770	5.3e-04	5.1e-04	3.2e-03	2.6e+00	3.7e+00	2.5e+00	2.7e+01

500 71 b4p2unix/1/14/46/ (a=25,b=2.5,c=-1.)

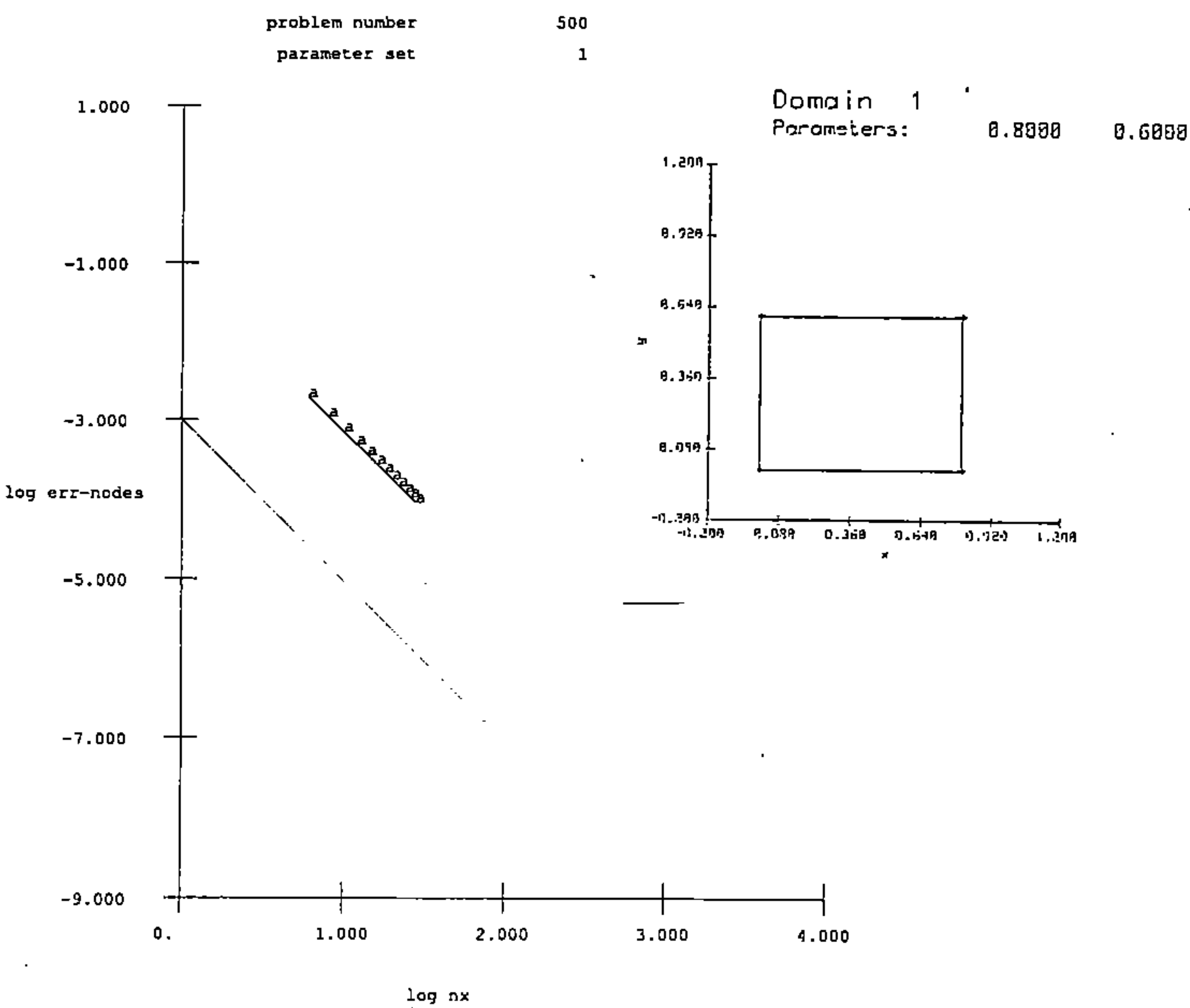
nx	ny	h	nunk	rerr	rerrmax	err12	conv2mx	conv2mm	conv212	solmax
7	7	3.5e-01	16	8.4e-03	1.0e+00	6.9e-02	0.0e+00	0.0e+00	0.0e+00	1.4e+01
11	11	2.1e-01	47	2.5e-03	9.6e-02	1.7e-02	2.4e+00	4.6e+00	2.7e+00	2.3e+01
13	13	1.7e-01	63	2.8e-03	5.8e-02	1.5e-02	1.6e+00	4.1e+00	2.2e+00	2.6e+01
15	15	1.5e-01	87	1.0e-03	2.3e-02	1.0e-02	2.5e+00	4.4e+00	2.3e+00	3.2e+01
17	17	1.3e-01	115	6.8e-04	1.6e-02	7.2e-03	2.6e+00	4.2e+00	2.3e+00	3.4e+01
19	19	1.2e-01	147	8.4e-04	3.6e-02	5.5e-03	2.1e+00	3.0e+00	2.3e+00	2.8e+01
21	21	1.0e-01	184	6.5e-04	9.5e-03	4.2e-03	2.1e+00	3.9e+00	2.3e+00	3.1e+01
25	25	8.7e-02	261	2.7e-04	2.3e-03	2.9e-03	2.5e+00	4.4e+00	2.3e+00	3.6e+01
29	29	7.4e-02	356	2.7e-04	3.6e-03	2.1e-03	2.2e+00	3.7e+00	2.3e+00	3.2e+01
55	55	3.8e-02	1314	4.6e-05	4.7e-04	5.8e-04	2.4e+00	3.5e+00	2.2e+00	3.5e+01
75	75	2.8e-02	2473	2.4e-05	3.5e-05	3.1e-04	2.3e+00	4.1e+00	2.2e+00	3.6e+01

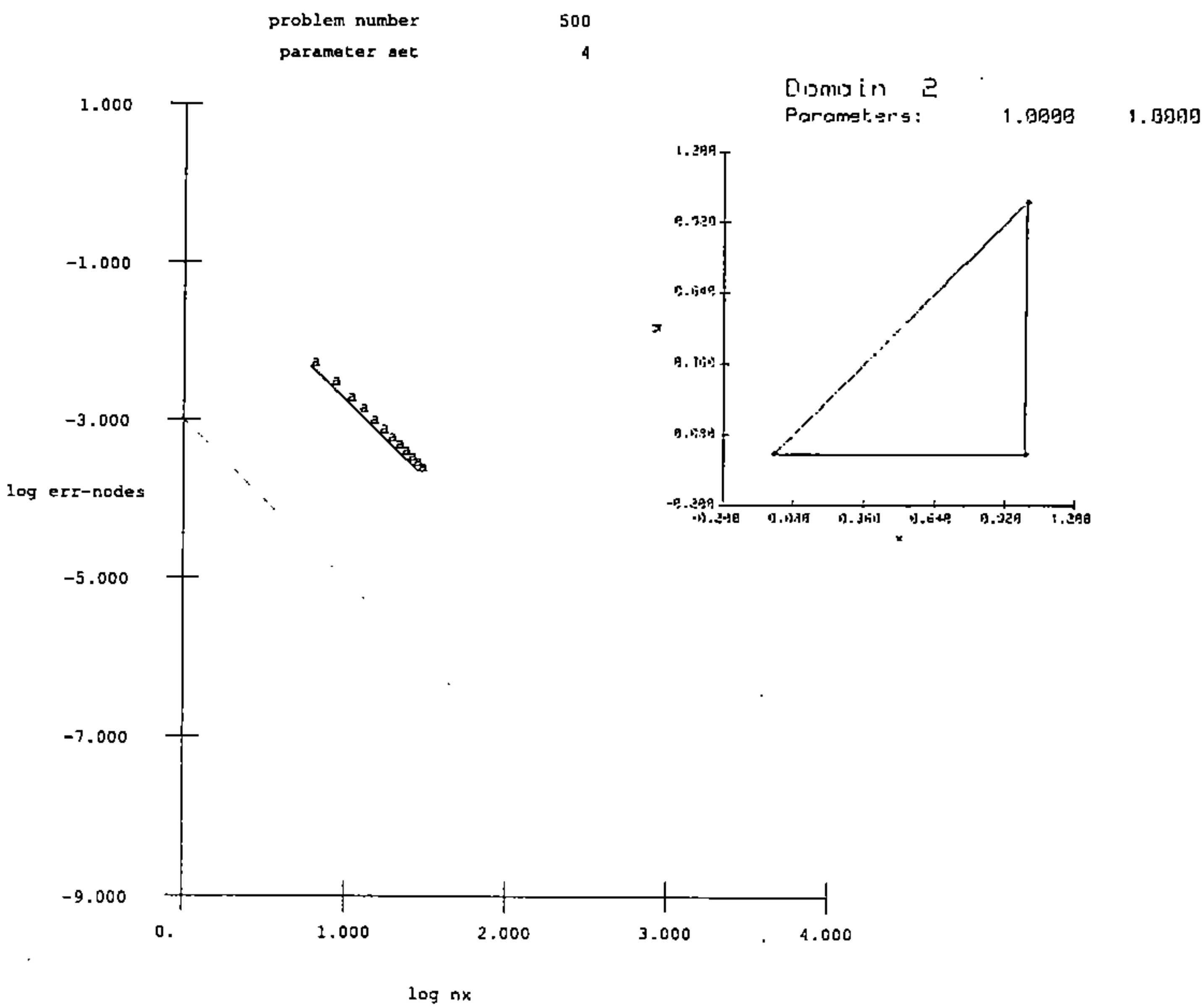
*** number of ellpack runs = 306

*** total CPU hours = 0.64

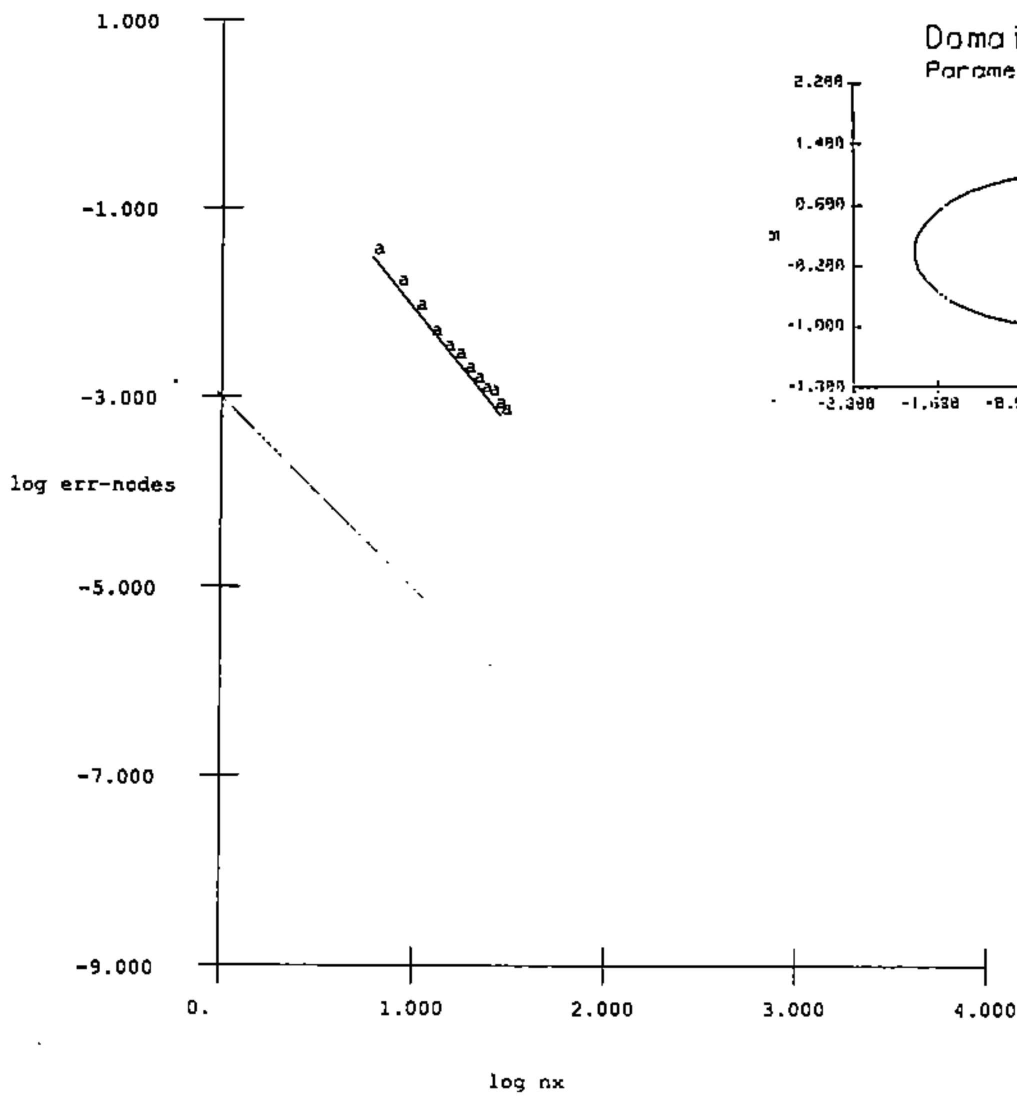
APPENDIX A. The Experimental Data

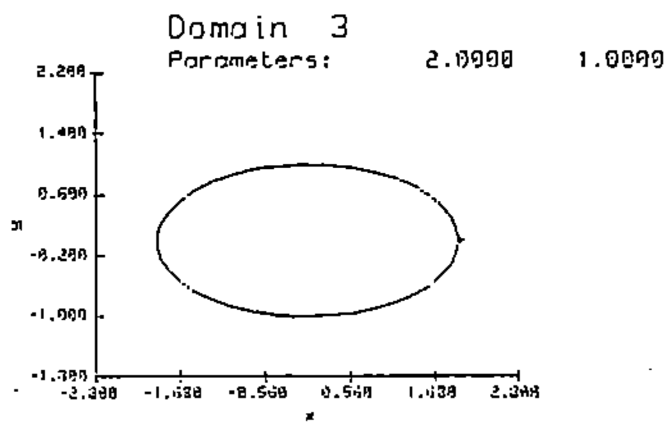
2. Domain shapes and convergence rate plots

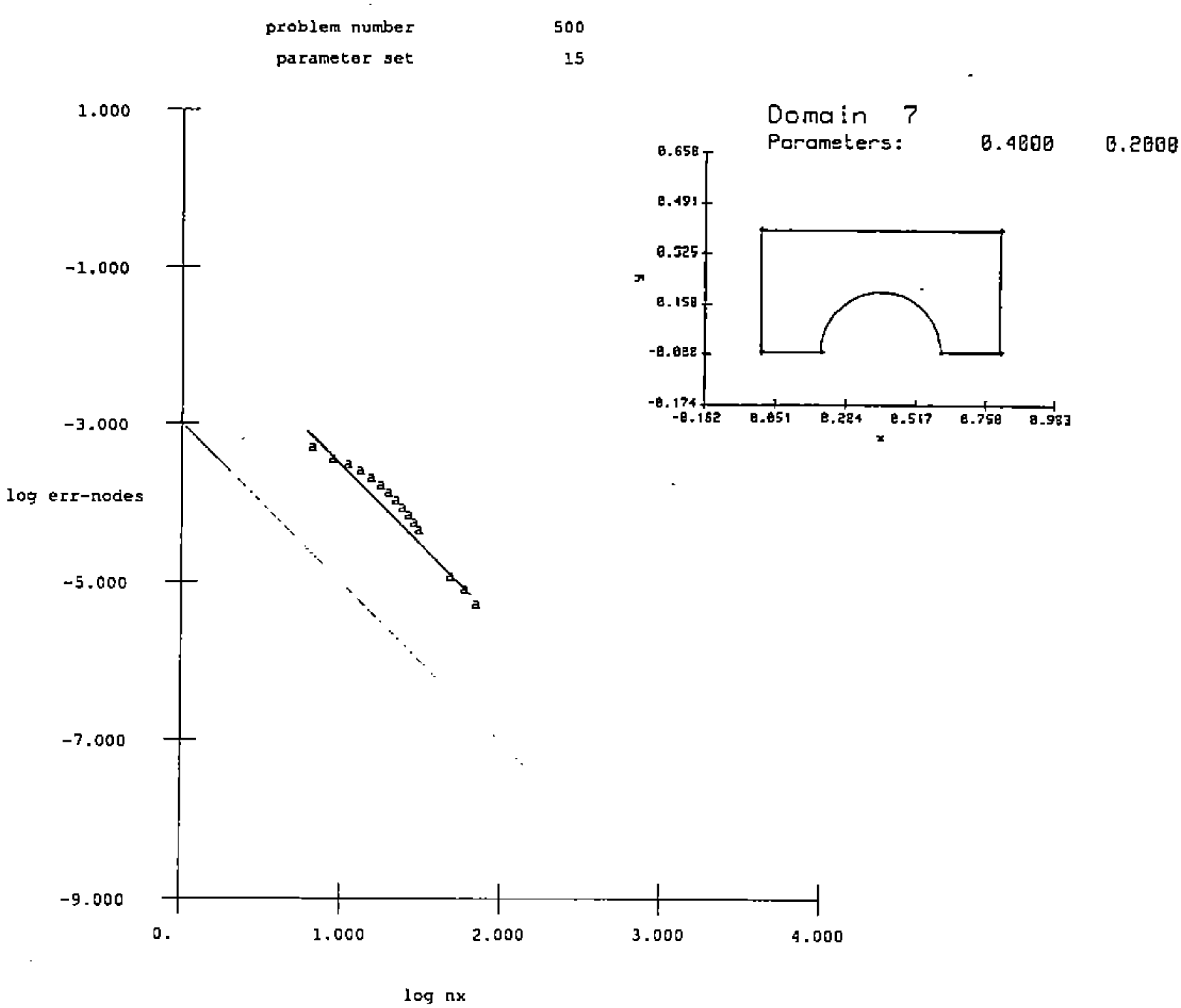



problem number 500
parameter set 4
1.000
-1.000
-3.000
-5.000
-7.000
-9.000
log err-nodes
0.
1.000
2.000
3.000
4.000
log nx
Domain 2
Parameters: 1.0000 1.0000

problem number 500
parameter set 5

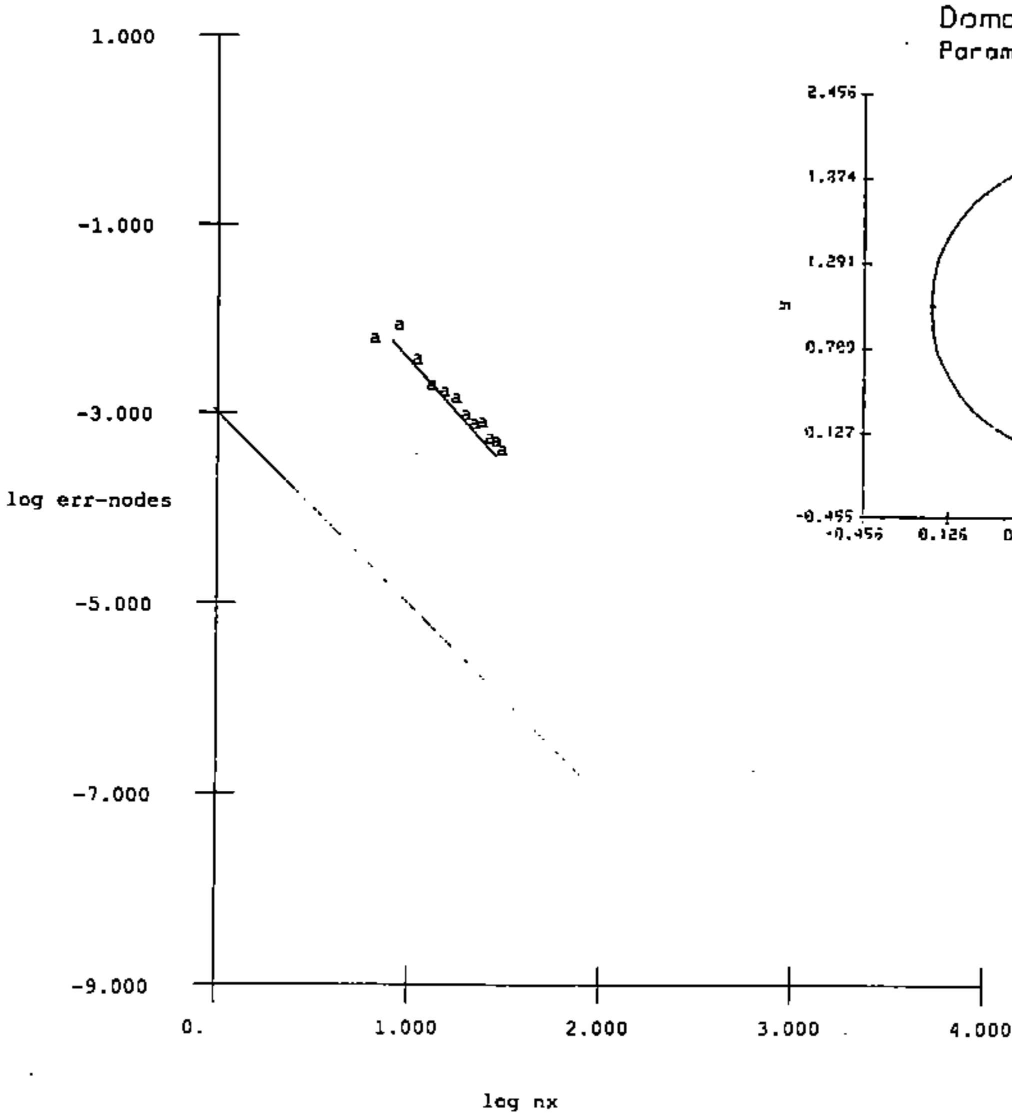


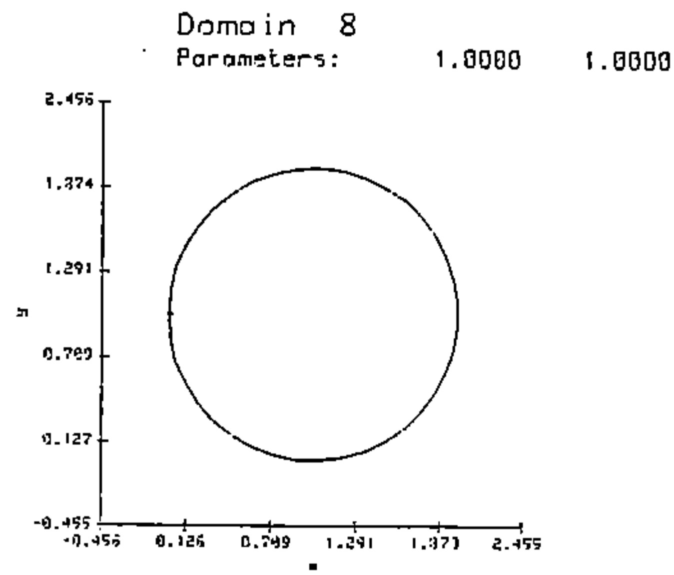


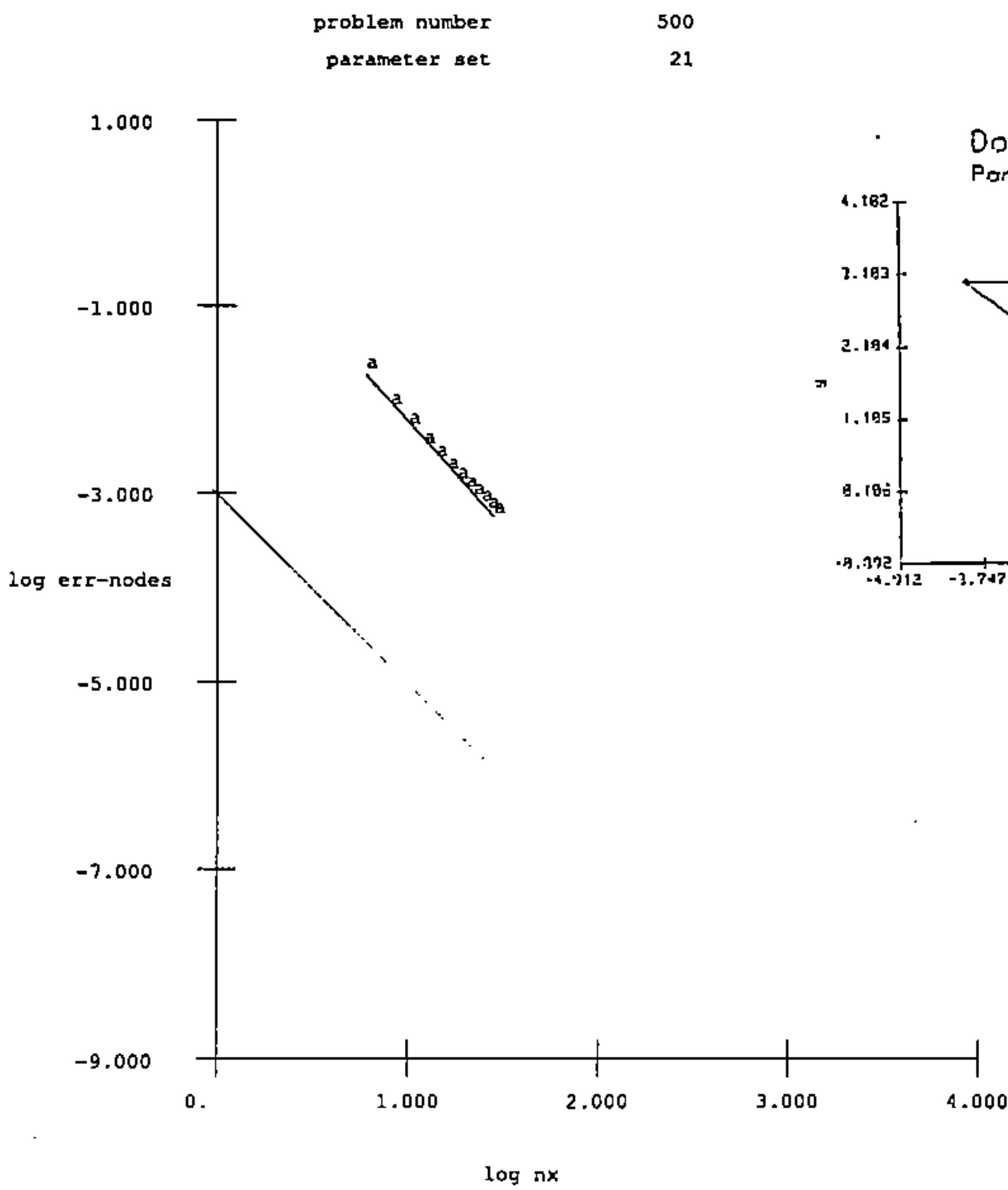

problem number 500
parameter set 15
1.000
-1.000
-3.000
-5.000
-7.000
-9.000
log err-nodes
0.
1.000
2.000
3.000
4.000
log nx
Domain 7
Parameters: 0.4000 0.2000
x
y

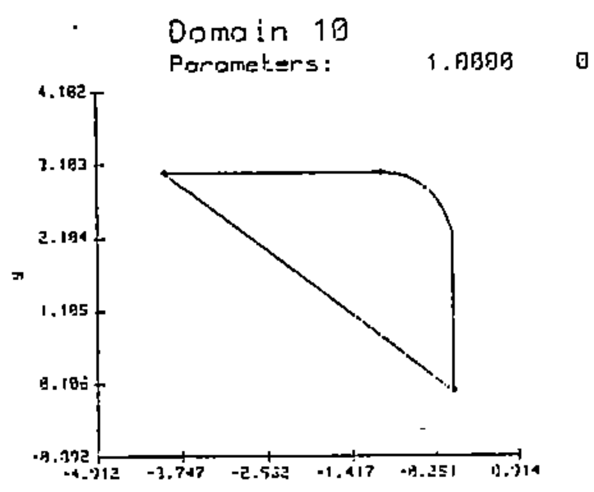
problem number 500

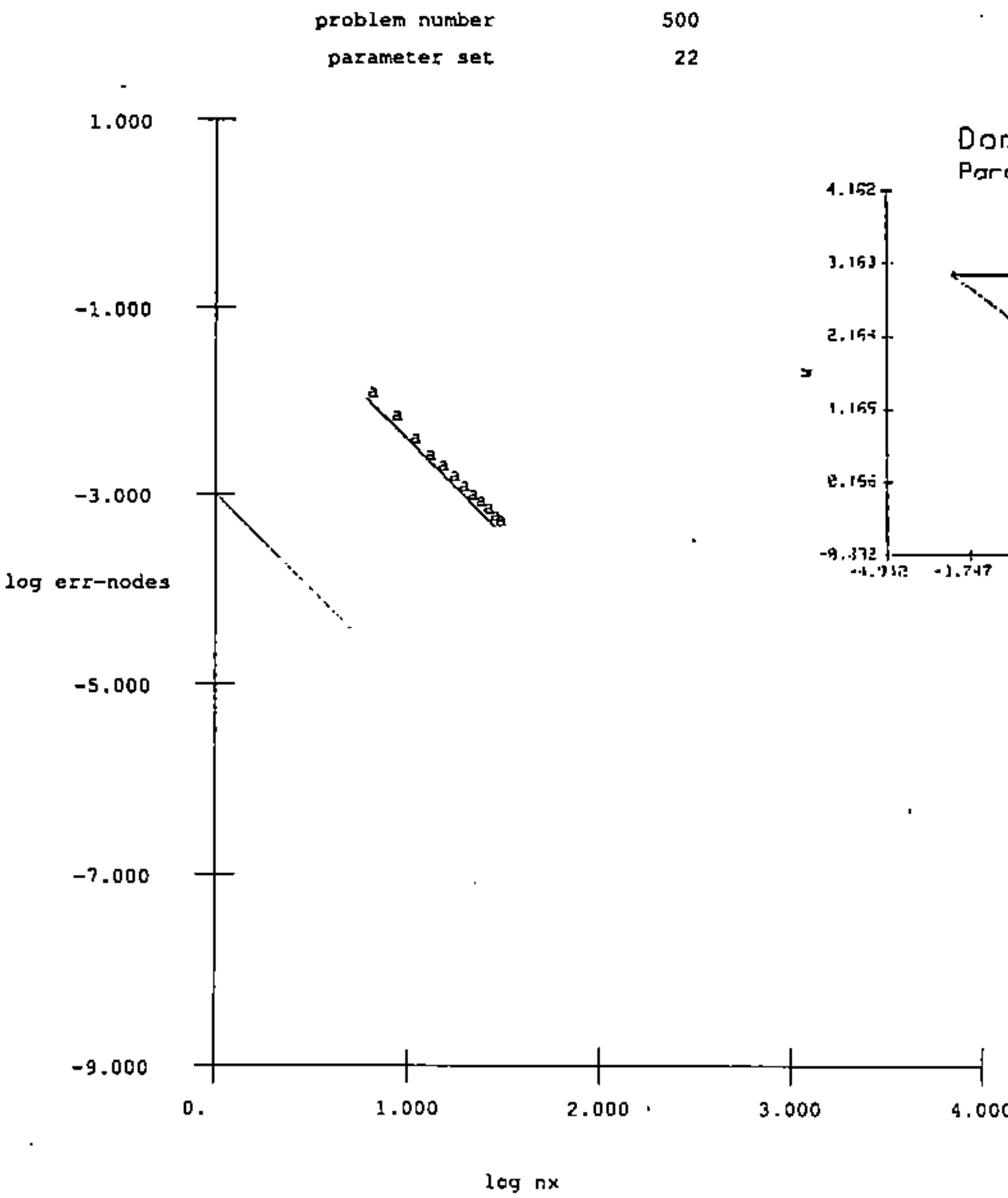
parameter set 17



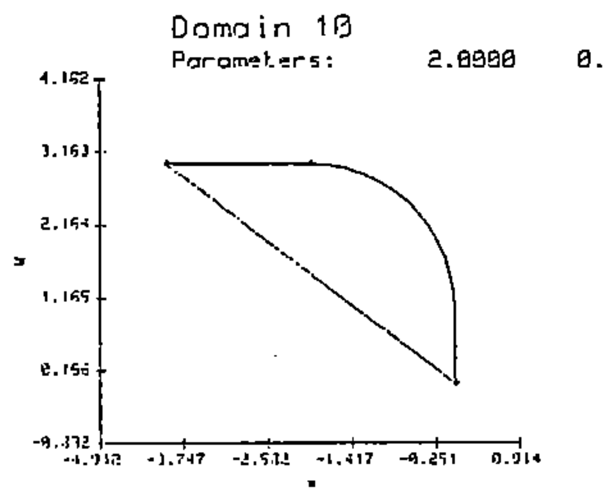



problem number 500
parameter set 21
1.000
-1.000
-3.000
-5.000
-7.000
-9.000
log err-nodes
0.
1.000
2.000
3.000
4.000
log nx


Domain 10
Parameters: 1.0000 0.



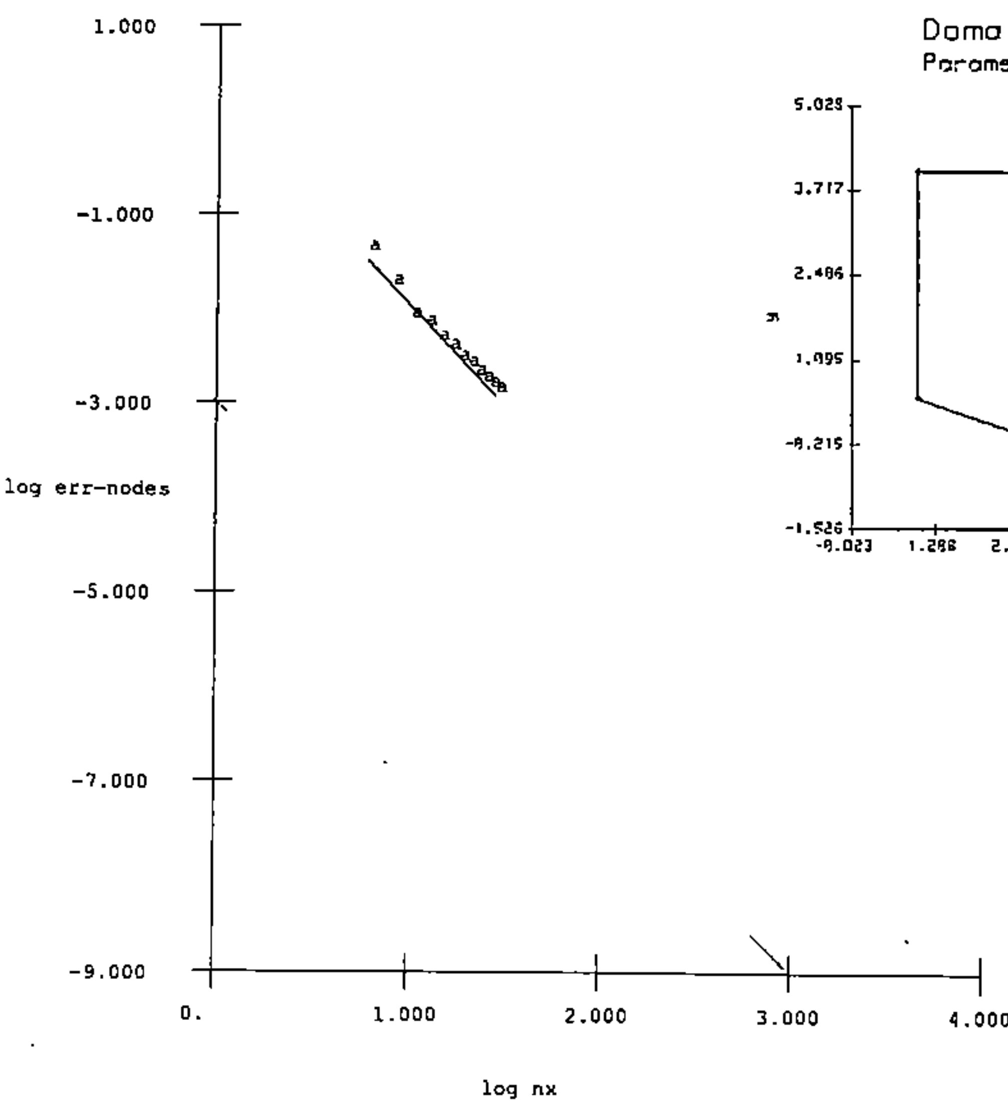
problem number 500
parameter set 22
1.000
-1.000
-3.000
-5.000
-7.000
-9.000
log err-nodes
0.
1.000
2.000
3.000
4.000
log nx



Domain 10
Parameters: 2.0000 0.

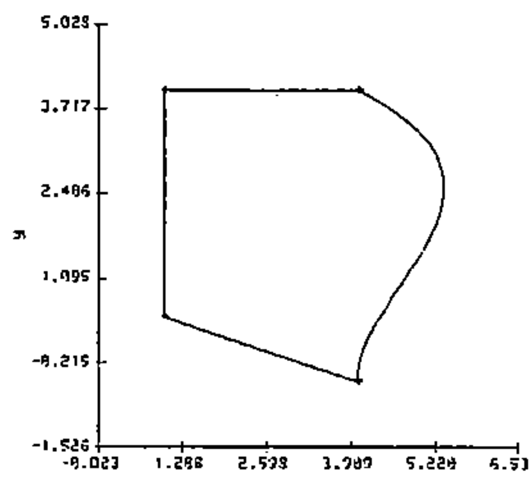
problem number 500

parameter set 25



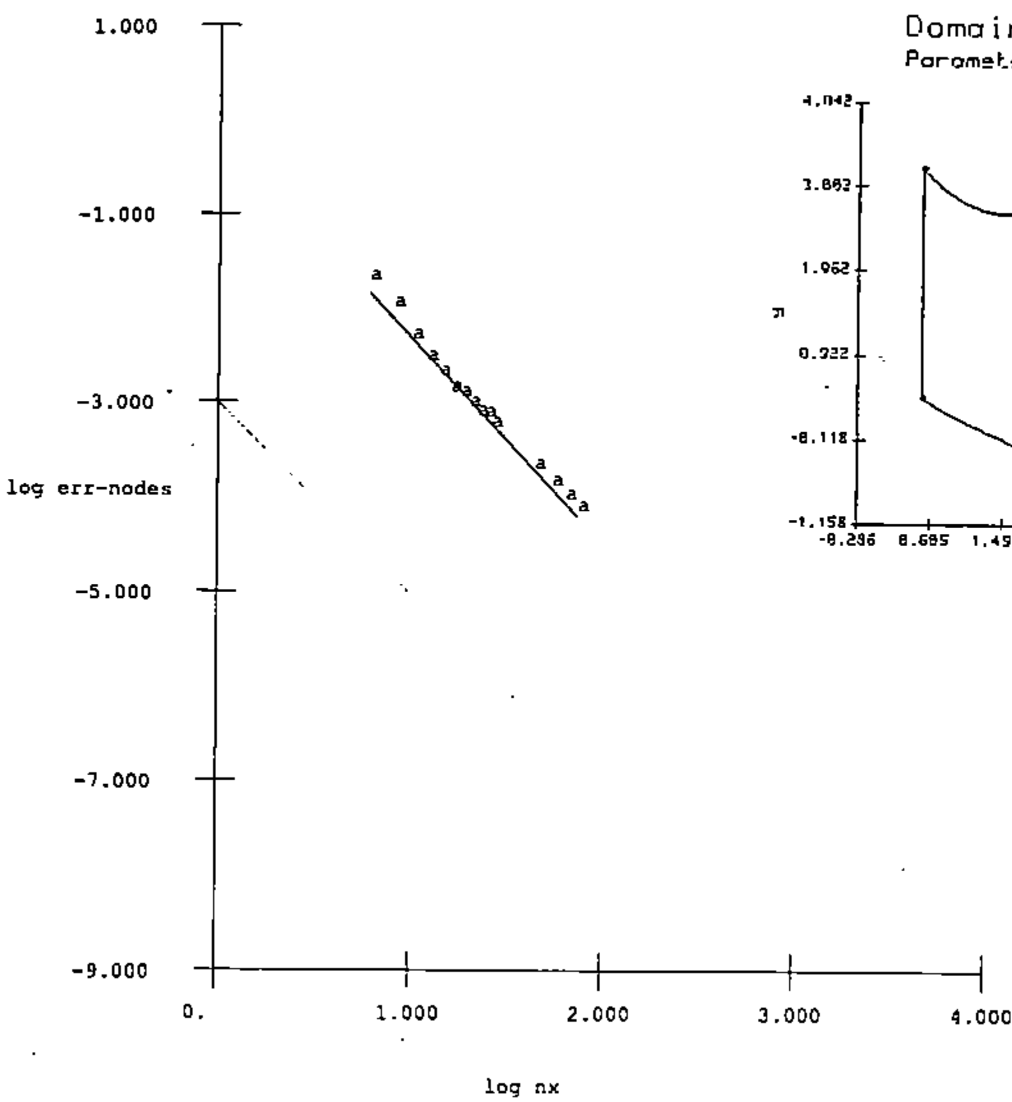
Domain 11

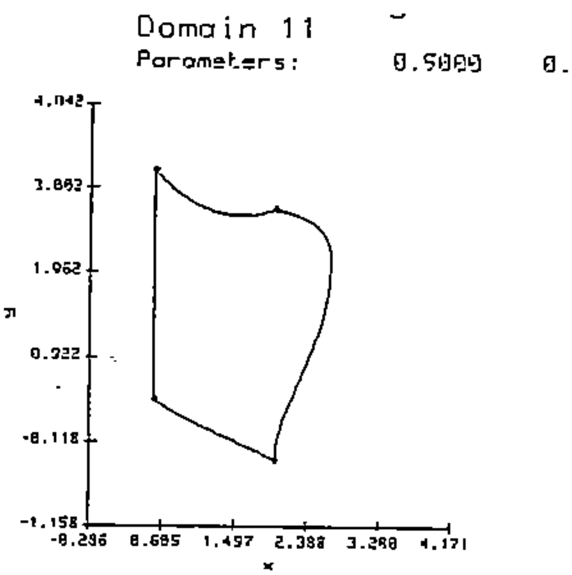
Parameters: 1.0000 0.

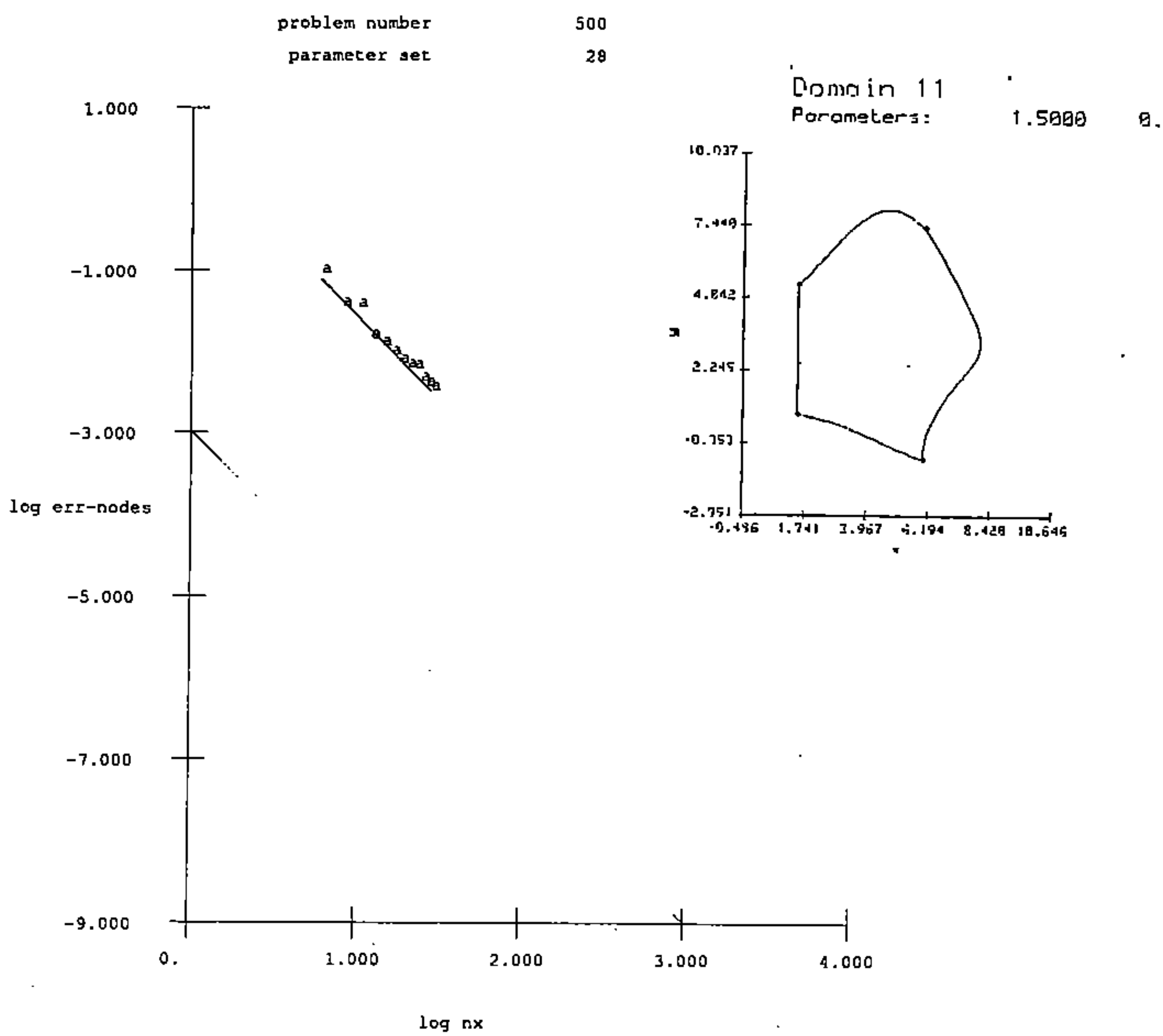


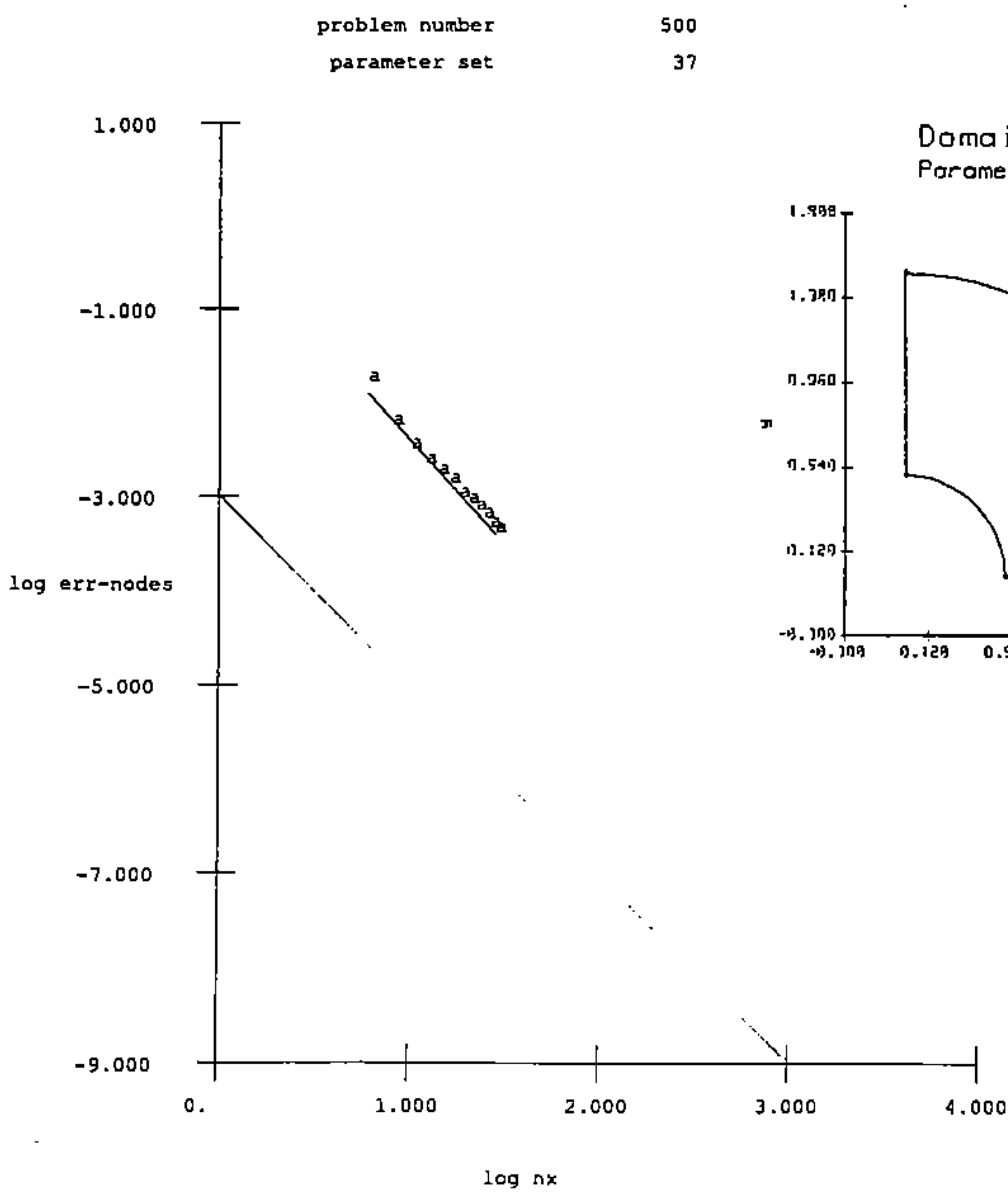
problem number 500

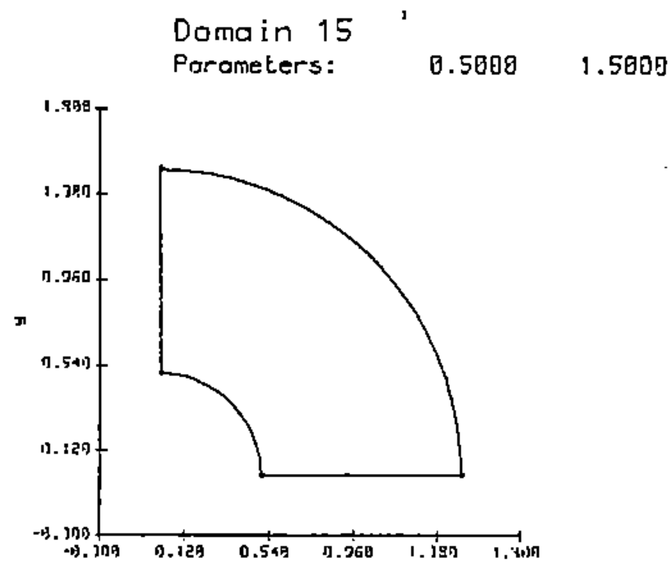
parameter set 27

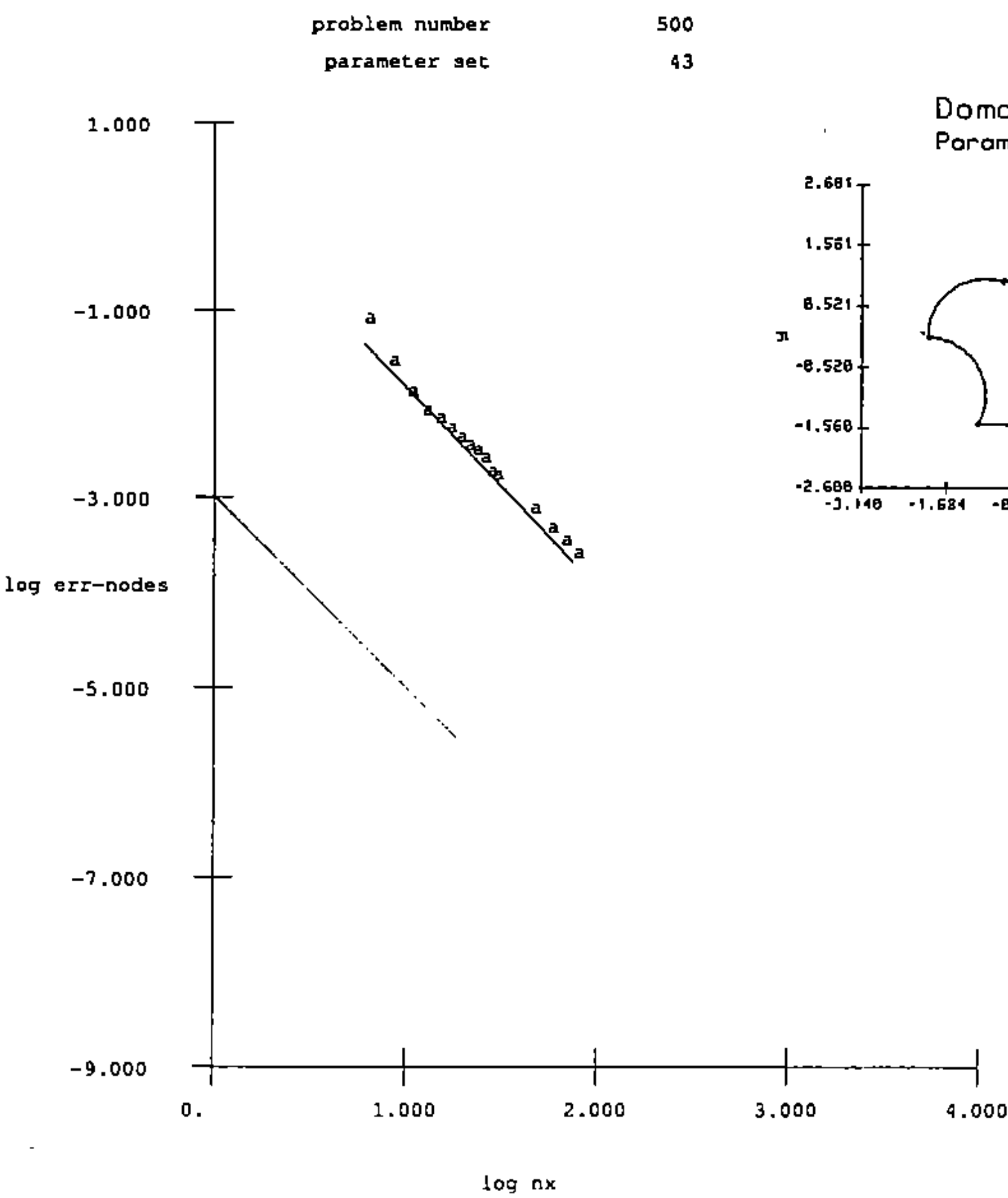


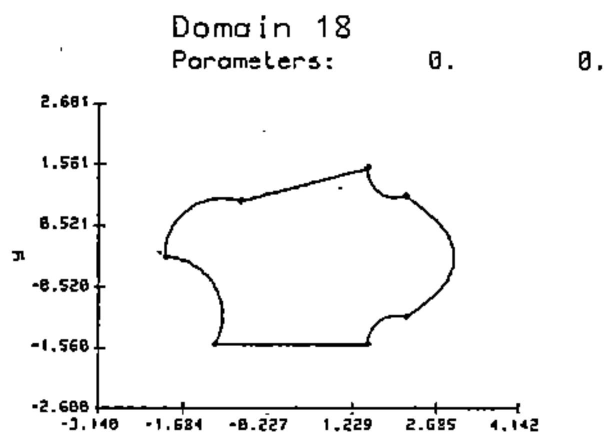


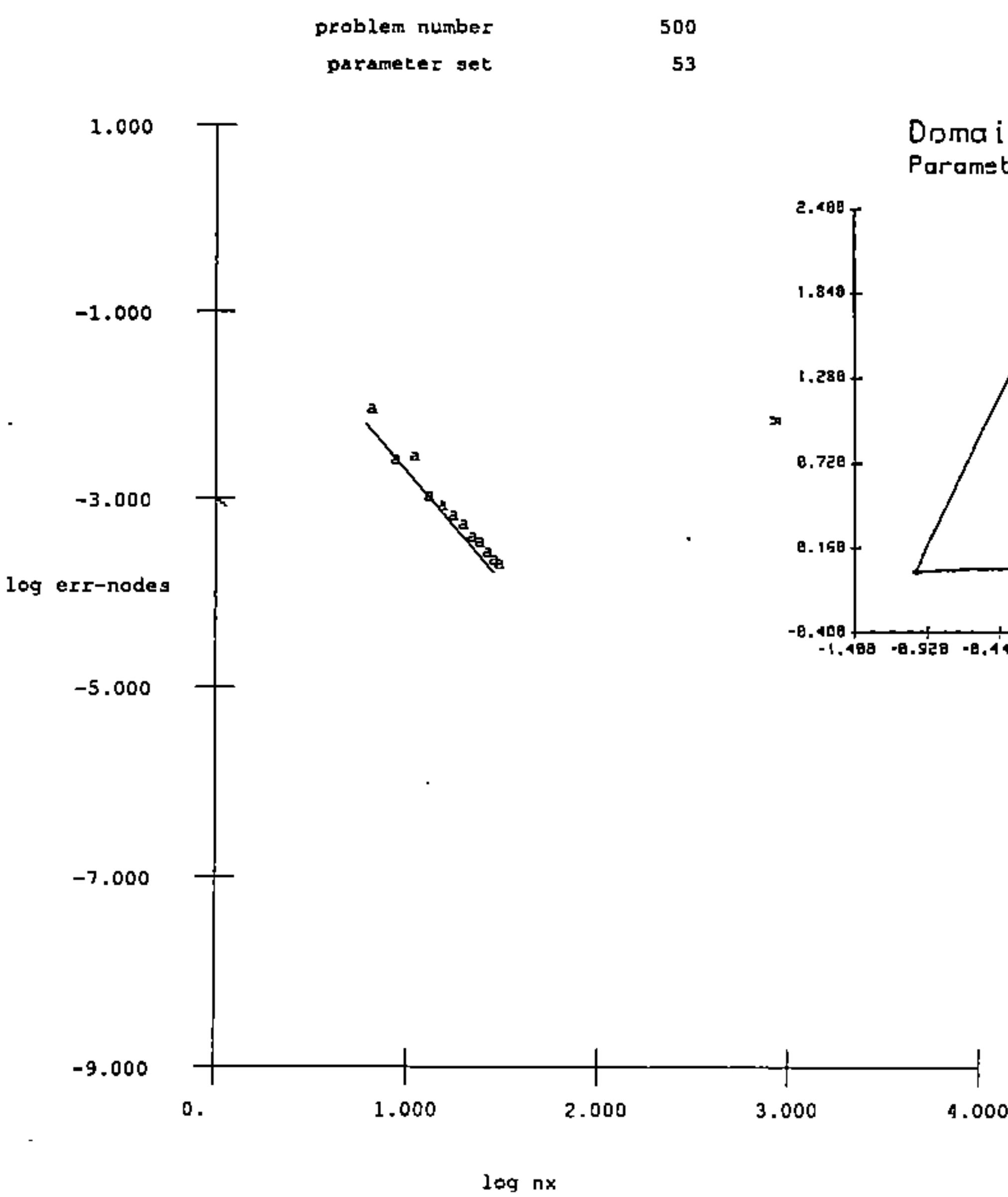

problem number 500
parameter set 28
1.000
-1.000
-3.000
-5.000
-7.000
-9.000
log err-nodes
0.
1.000
2.000
3.000
4.000
log nx
Domain 11
Parameters: 1.5000 0.

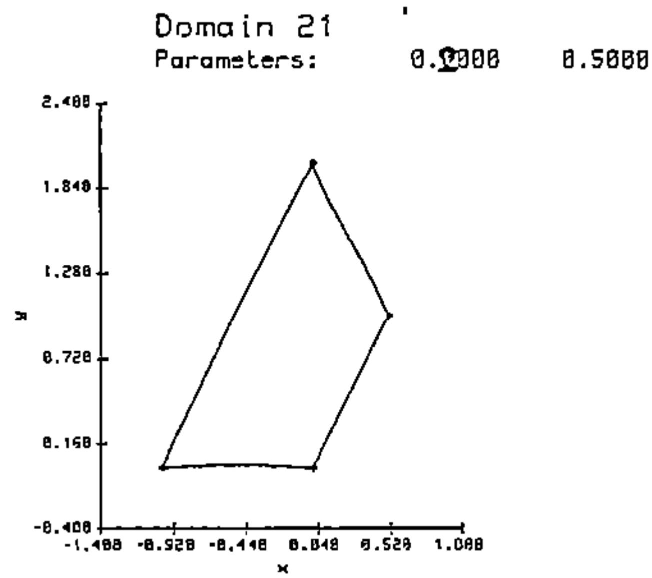

problem number 500
parameter set 37
1.000
-1.000
-3.000
-5.000
-7.000
-9.000
log err-nodes
0.
1.000
2.000
3.000
4.000
log nx
a


Domain 15
Parameters: 0.5000 1.5000


problem number 500
parameter set 43
1.000
-1.000
-3.000
-5.000
-7.000
-9.000
log err-nodes
0.
1.000
2.000
3.000
4.000
log nx


Domain 18
Parameters: 0. 0.


problem number 500
parameter set 53
1.000
-1.000
-3.000
-5.000
-7.000
-9.000
log err-nodes
0.
1.000
2.000
3.000
4.000
log nx

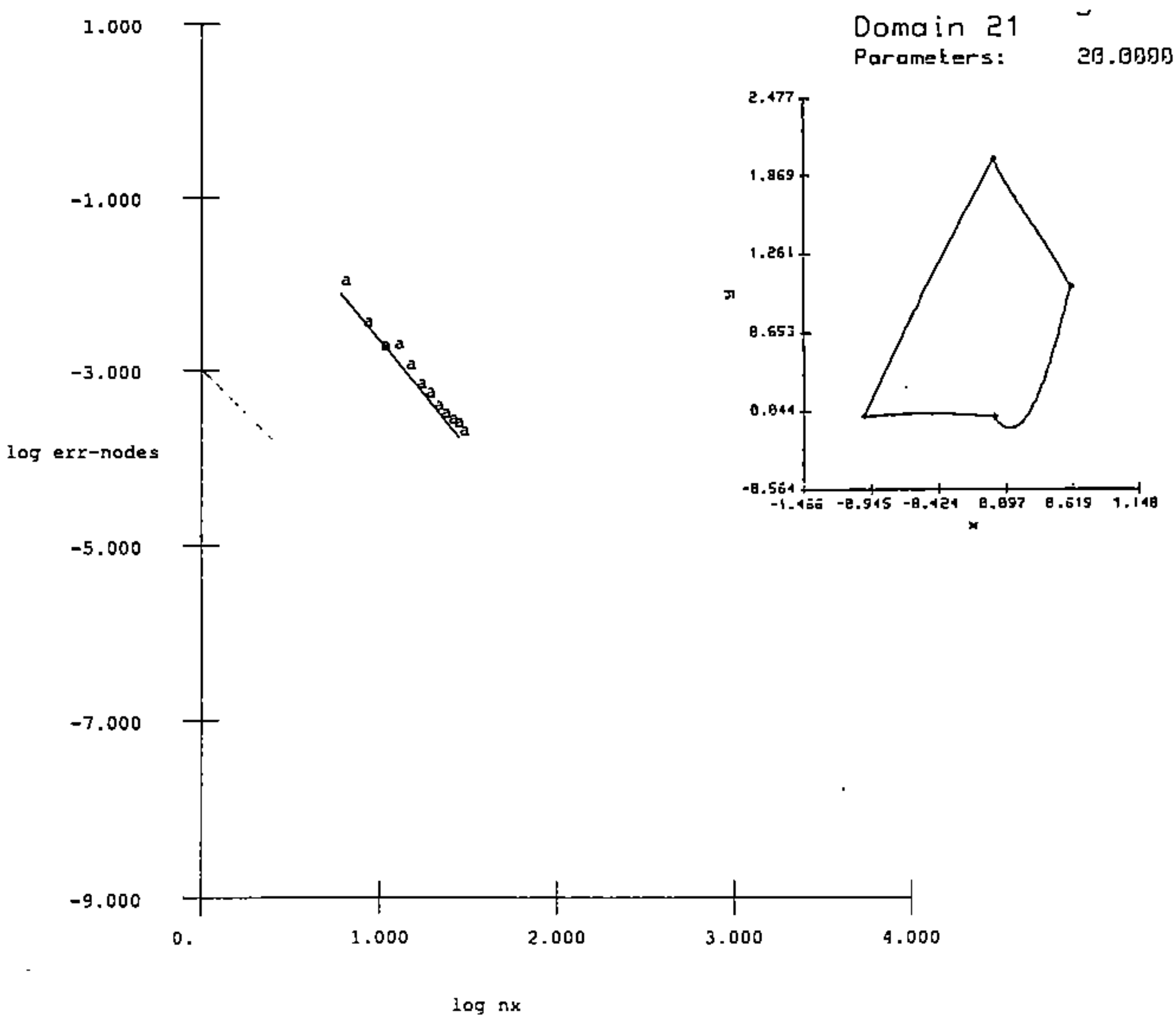

Domain 21
Parameters:
0.5000
2.400
1.840
1.280
0.720
0.160
-0.400
y
-1.400
-0.920
-0.440
0.040
0.520
1.000
x

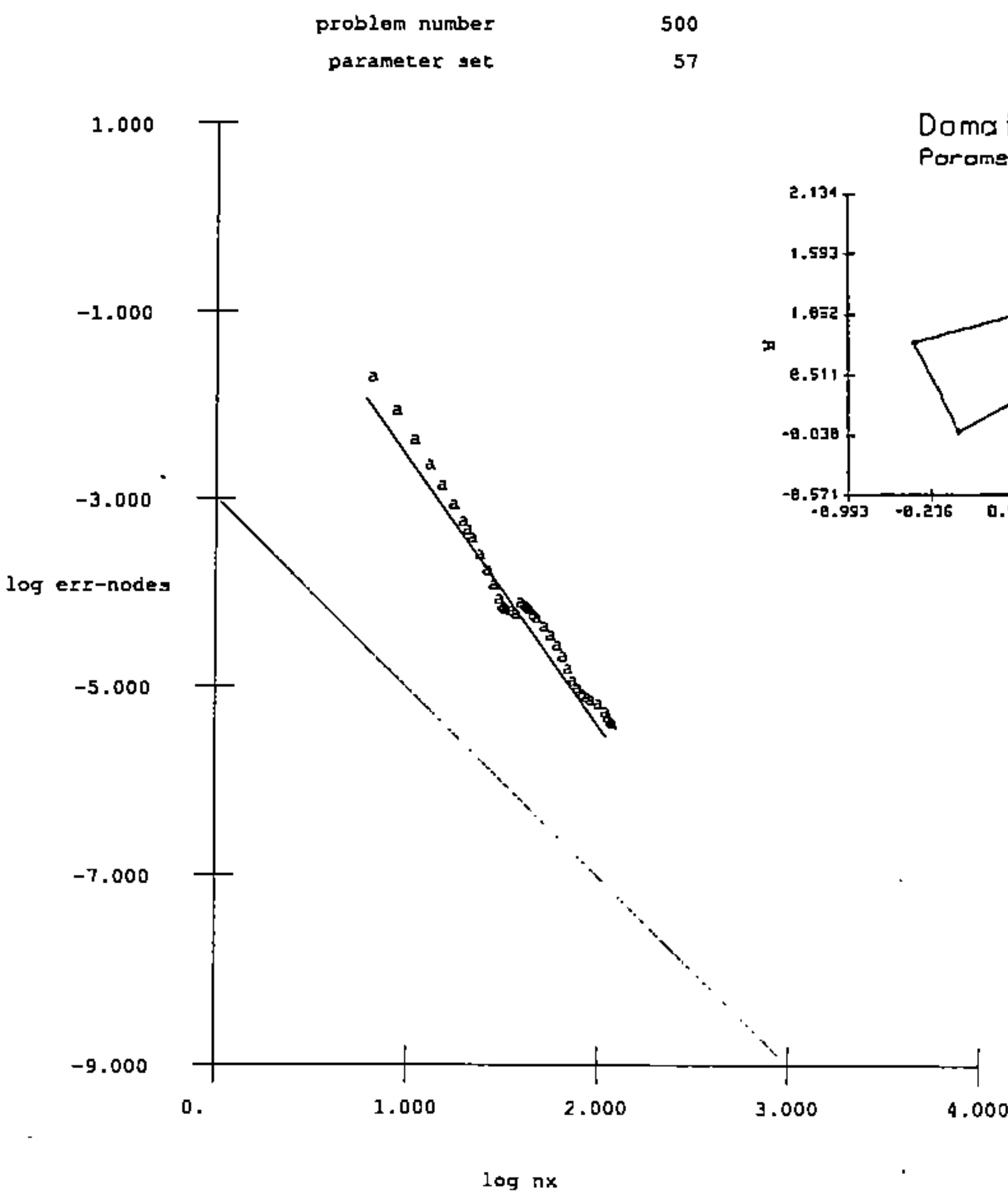
problem number 500

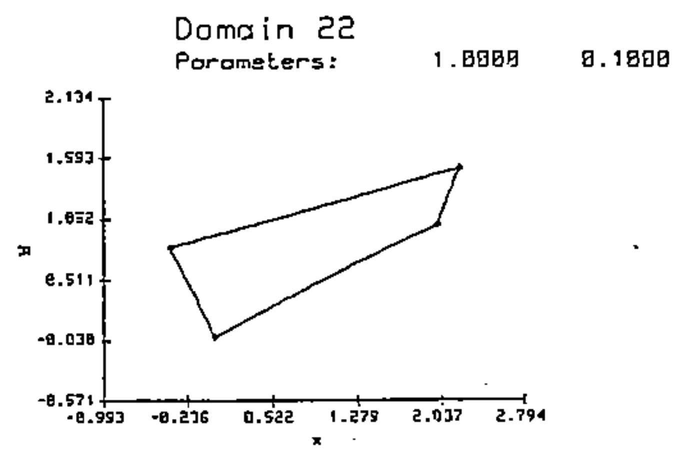
parameter set 55

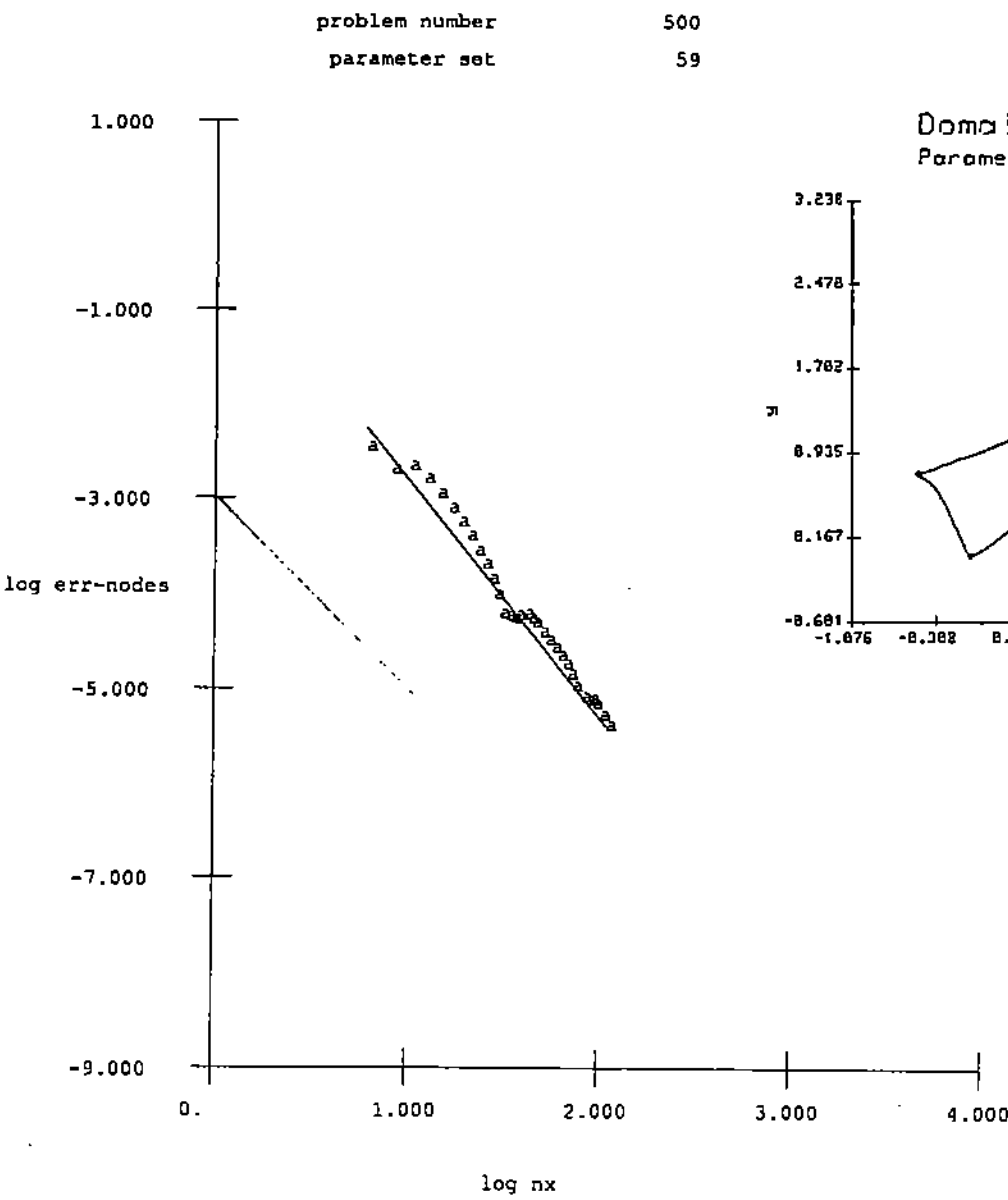
Domain 21

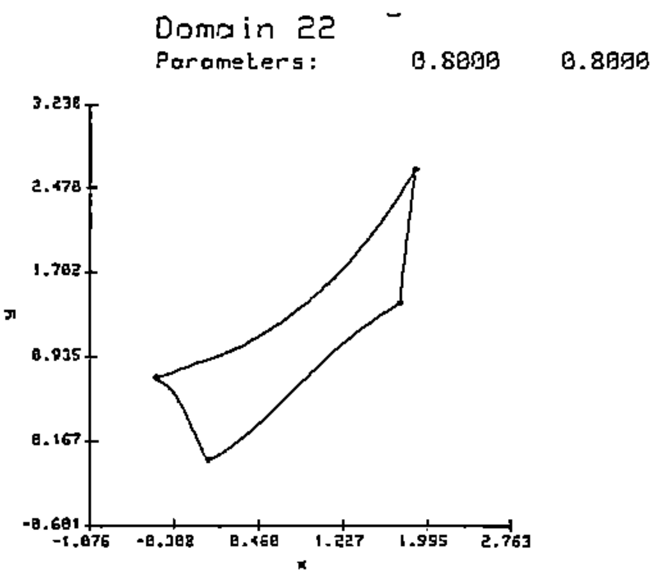
Parameters: 20.0000 0.6000

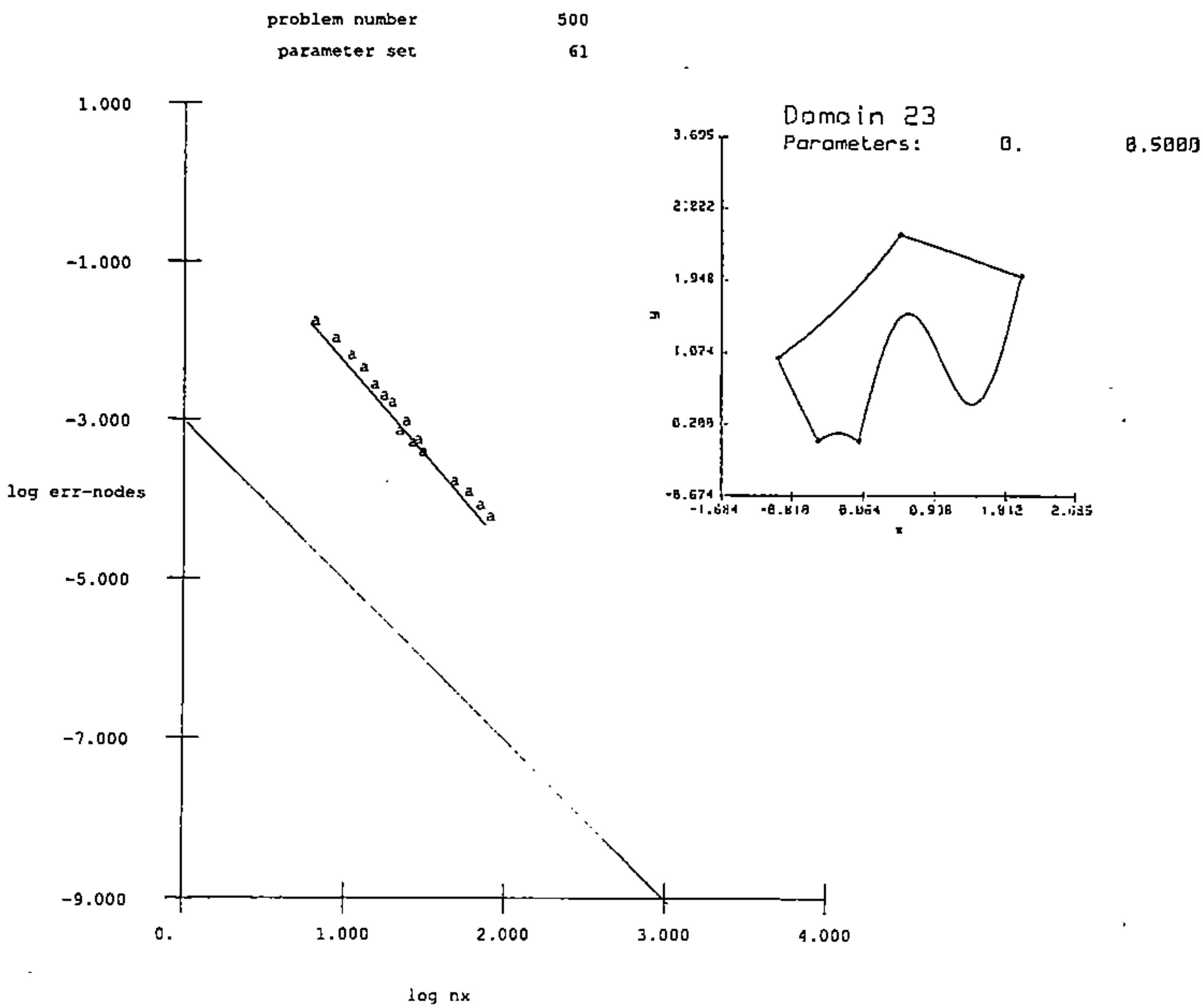


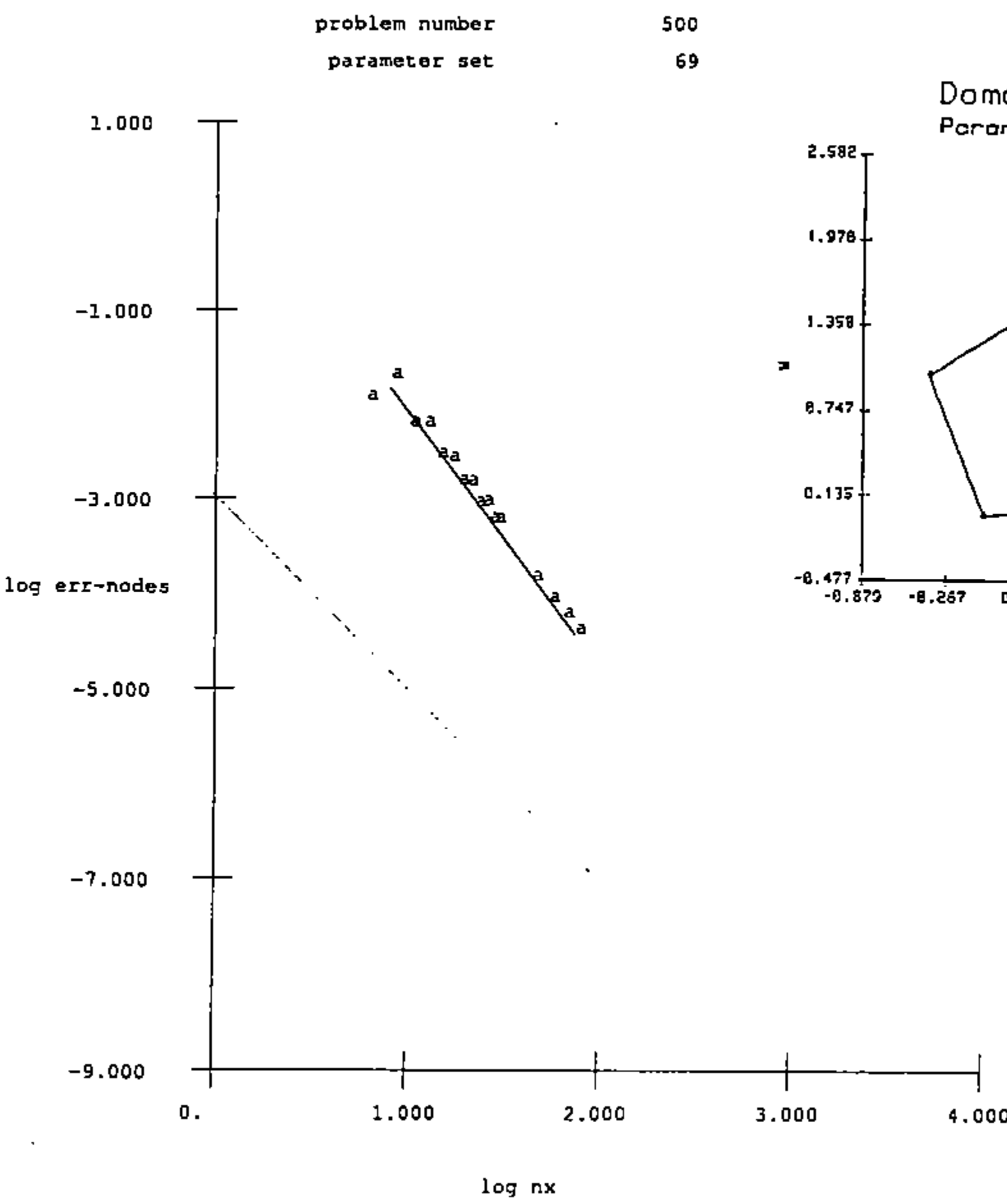

problem number 500
parameter set 57
1.000
-1.000
-3.000
-5.000
-7.000
-9.000
log err-nodes
0.
1.000
2.000
3.000
4.000
log nx

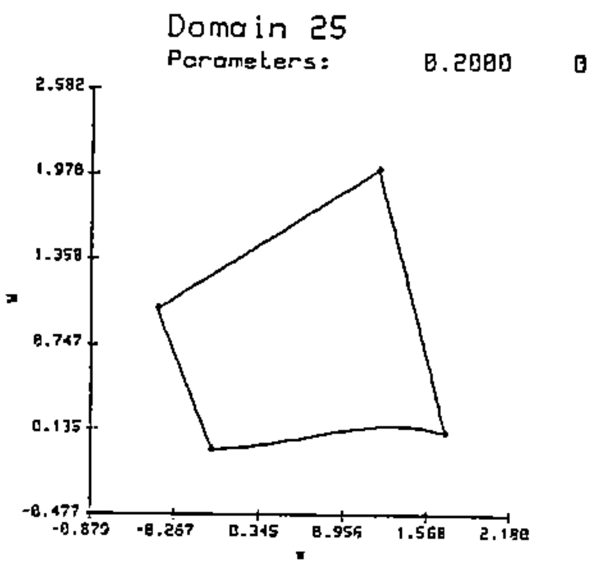

Domain 22
Parameters: 1.0000 0.1000
2.134
1.593
1.052
0.511
-0.030
-0.571
-0.993
-0.236
0.522
1.279
2.037
2.794
x
y

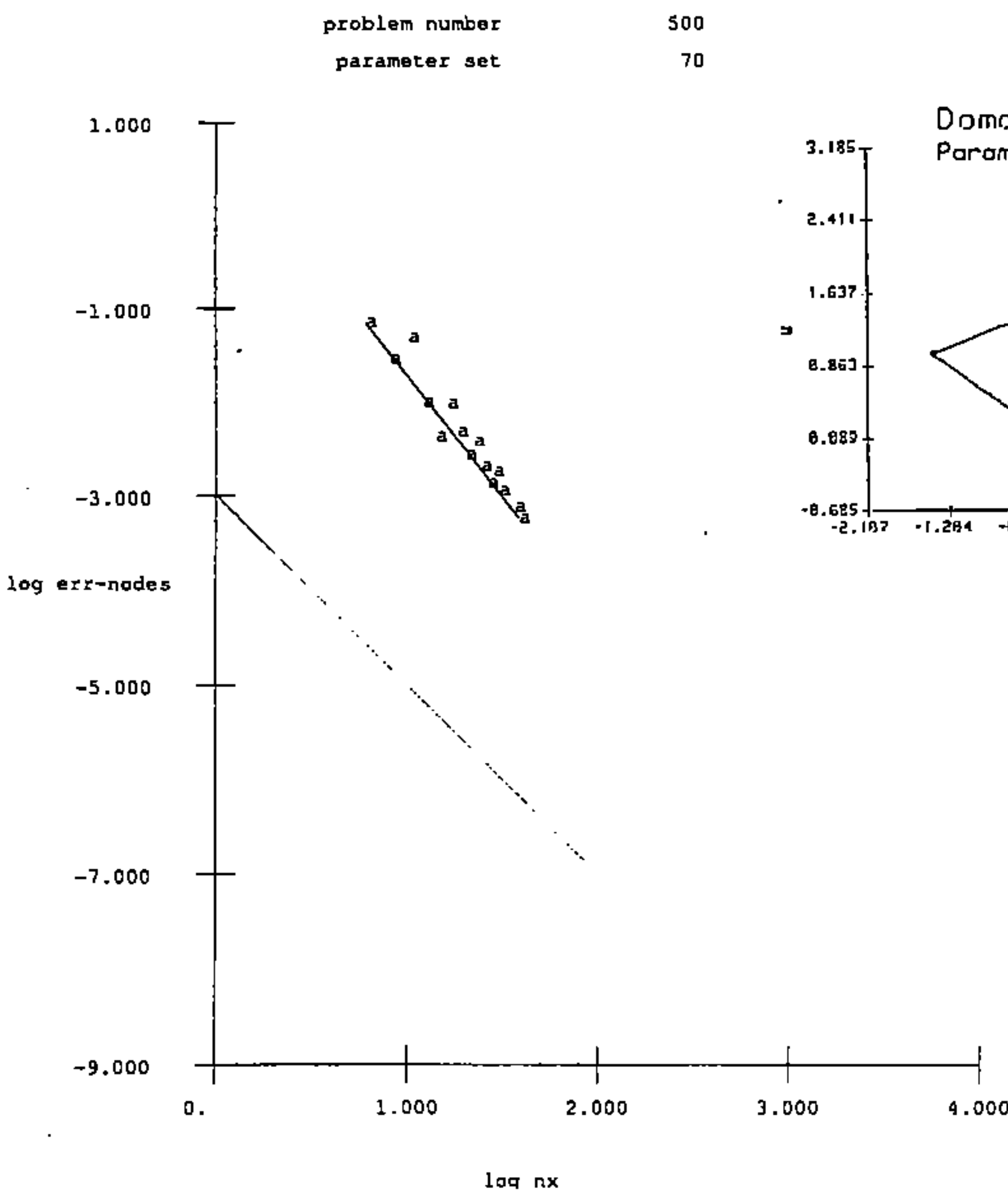

problem number 500
parameter set 59
1.000
-1.000
-3.000
-5.000
-7.000
-9.000
log err-nodes
0.
1.000
2.000
3.000
4.000
log nx

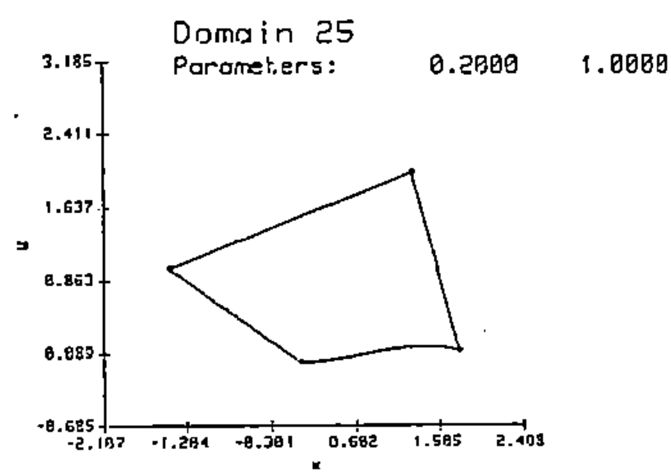

Domain 22
Parameters: 0.8000 0.8000
y
x

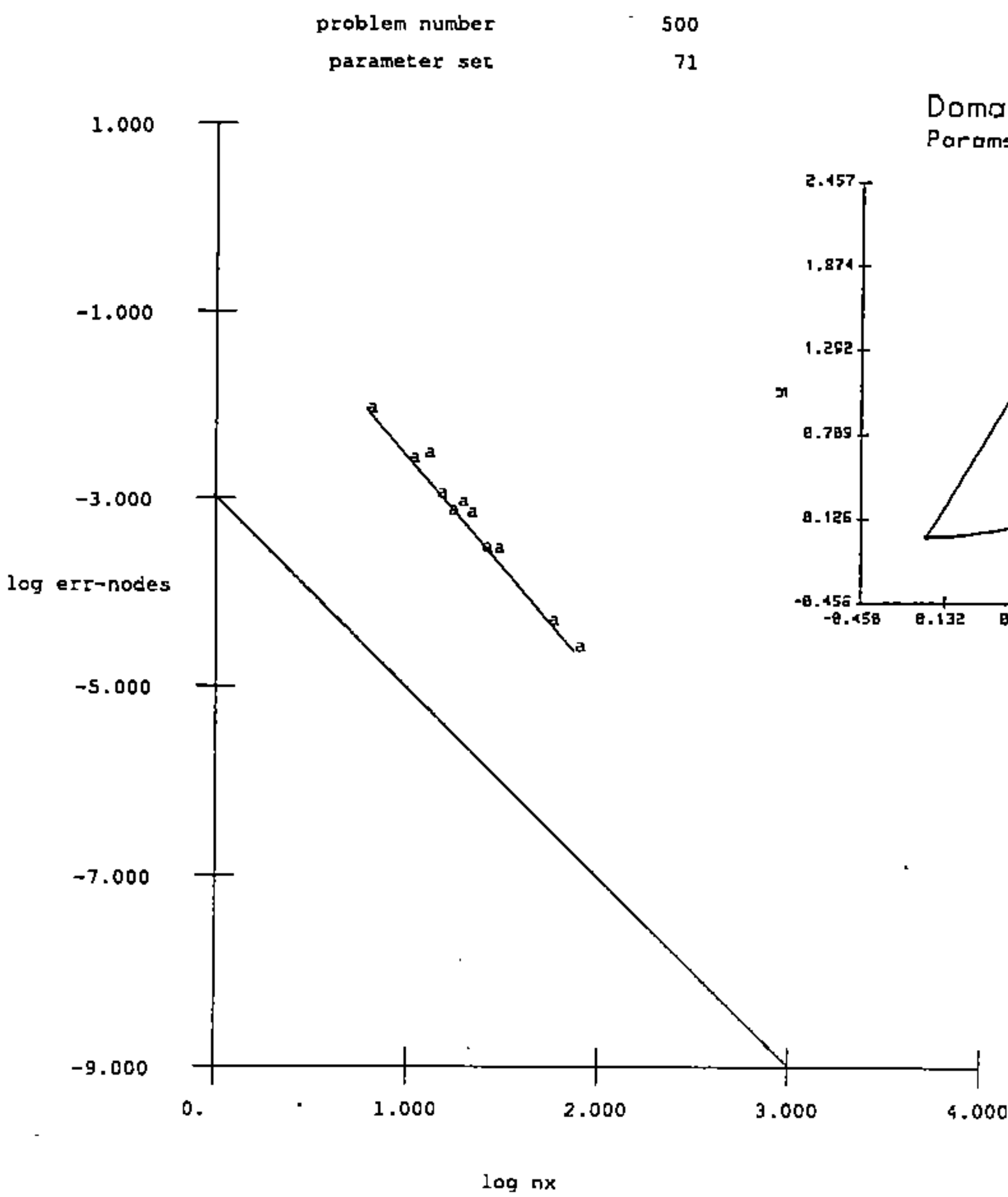

problem number 500
parameter set 61
1.000
-1.000
-3.000
log err-nodes
-5.000
-7.000
-9.000
0.
1.000
2.000
3.000
4.000
log nx
Domain 23
Parameters: 0. 0.5000


problem number 500
parameter set 69
1.000
-1.000
-3.000
-5.000
-7.000
-9.000
log err-nodes
0.
1.000
2.000
3.000
4.000
log nx

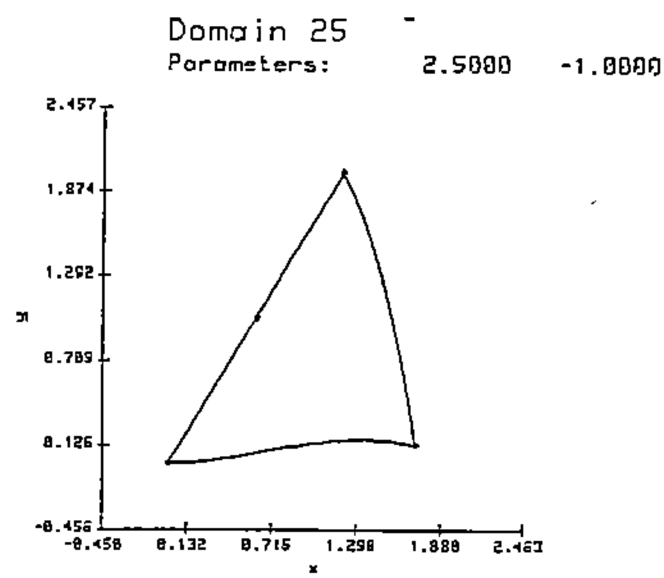

Domain 25
Parameters: 0.2000 0.
2.582
1.970
1.358
0.747
0.135
-0.477
-0.879
-0.267
0.345
0.956
1.568
2.180


problem number 500
parameter set 70
1.000
-1.000
-3.000
-5.000
-7.000
-9.000
log err-nodes
0.
1.000
2.000
3.000
4.000
log nx


Domain 25
Parameters: 0.2000 1.0000



problem number 500
parameter set 71
1.000
-1.000
-3.000
log err-nodes
-5.000
-7.000
-9.000
0.
1.000
2.000
3.000
4.000
log nx



Domain 25
Parameters: 2.5000 -1.0000
2.457
1.874
1.292
0.709
0.126
-0.456
-0.458
0.132
0.715
1.298
1.880
2.463
x